

**Student Code Number:** \_\_\_\_\_

# **Ph.D. Qualifying Exam**

## **Fluid Mechanics**

**Spring 2010**

**Prof. F. Jaberı  
Prof. N. Priezjev**

**Directions: Closed Book but formula sheet is provided**

**Answer all four questions**

**All questions have equal weight**

**Time: 3.0 hrs.**

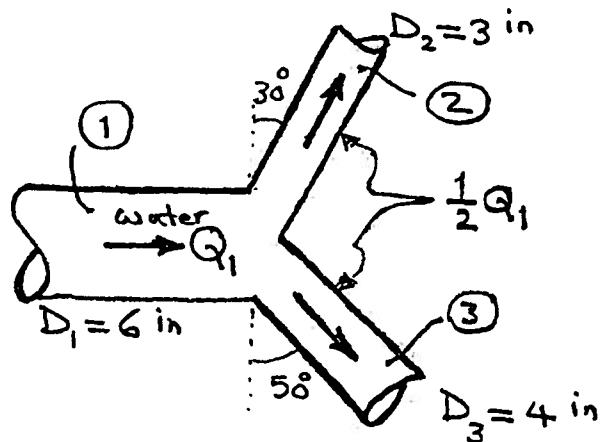
.....

- Take any required property from your book, approximate values if necessary.
- If you make any assumption to reach a solution state it clearly

.....

### Problem 1

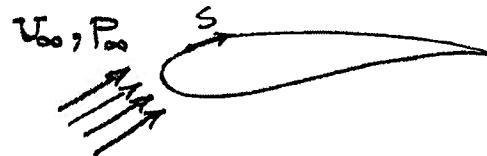
Water at room temperature is divided equally into two steady streams in the following fitting system. For flow rate of  $Q_1 = 5 \text{ ft}^3/\text{s}$  and pressure of  $p_1 = 25 \text{ lbf/in}^2$  (gage), estimate (a)  $P_2$  (b)  $p_3$ , (c) Vector force required to keep the fitting in place. Neglect all losses and gravity. Density for water in room temperature is  $1.94 \text{ slug/ft}^3$ . You may assume the flow to be steady and inviscid.



## Problem 2

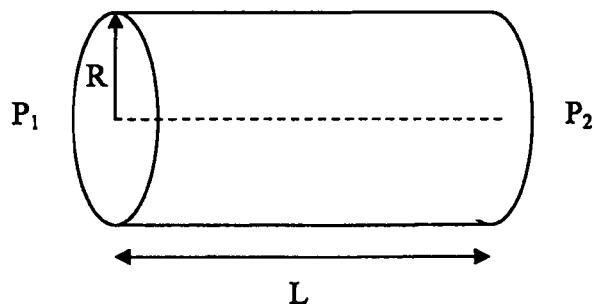
Consider the flow over a two-dimensional airfoil. Answer the following questions:

- (a) Show the direction of lift and drag force on the following figure. How does the airfoil generate lift? Explain.
- (b) Plot velocity and pressure variations on the lower and upper surface of the airfoil as a function of  $s$ .
- (c) Explain how you calculate the lift and drag forces on the airfoil for a given velocity distribution (i.e. velocity vector at all spatial positions are known). Assume the flow to be inviscid and steady.
- (d) List main differences between the inviscid and viscous flows over the airfoil. What is the effect of viscosity on the drag?
- (e) Where and why do you get boundary layer separation? What are the effects of separation on lift and drag? What is the effect of free stream turbulence on the separation?



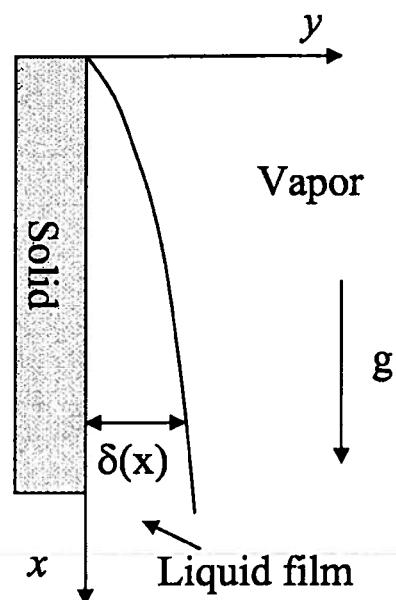
### Problem 3

Consider a fully developed flow in a circular pipe as shown in the figure. Assume steady axisymmetric laminar flow without any swirl ( $u_\theta = 0$ ) and  $L$  is larger than  $R$ . Pressure  $P_1$  on the left side of the pipe is larger than  $P_2$  on the right side.  $\mu$  is the fluid viscosity and  $\rho$  is the fluid density. Derive the expression for the velocity profiles  $u(r)$  as a function of the distance  $r$  from the axis for two cases: (a) no-slip boundary condition at the wall and (b) partial slip boundary condition at the inner pipe wall  $-\partial u(r)/\partial r|_{r=R} \cdot L_s = u(r = R)$ . What is the ratio between mass flow rates for these two cases if  $L_s = 0.1 \cdot R$ ?



#### Problem 4

A thin liquid film is formed on a vertical surface due to vapor condensation (as shown in the figure). The film moves under the action of gravity and forms a laminar boundary layer. Treat the problem as two-dimensional, steady, and assume that the liquid film velocity component in the  $y$ -direction is negligible. Also, neglect the viscosity of the vapor (which is much smaller than the liquid viscosity). Use these simplifications to reduce the continuity and momentum equations and derive the velocity profile  $u(y)$  in the liquid film as a function of the film thickness  $\delta(x)$ . Find the mass flow rate in the film in terms of liquid density  $\rho$ , liquid kinematic viscosity  $\nu$ , and film thickness  $\delta(x)$ .



Integral mass conservation equation:

$$0 = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot d\vec{A}$$

Integral momentum equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \vec{V} \rho \vec{V} \cdot d\vec{A}$$

### The Equation of Continuity

---

Rectangular Coordinates ( $x, y, z$ ):

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates ( $r, \theta, z$ ):

$$\frac{\partial p}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

### Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity ( $\mu$ )

---

Rectangular Coordinates ( $x, y, z$ ):

$$\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left[ \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates ( $r, \theta, z$ ):

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_r^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] - \frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates ( $x, y, z$ )	Cylindrical Coordinates ( $r, \theta, z$ )
$\tau_{xx} = \mu \left[ 2 \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{rr} = \mu \left[ 2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{yy} = \mu \left[ 2 \frac{\partial v_y}{\partial y} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{\theta\theta} = \mu \left[ 2 \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{zz} = \mu \left[ 2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{zz} = \mu \left[ 2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$
$\tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu \left[ \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$
$\tau_{zx} = \tau_{xz} = \mu \left[ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right]$	$\tau_{rz} = \tau_{zr} = \mu \left[ \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

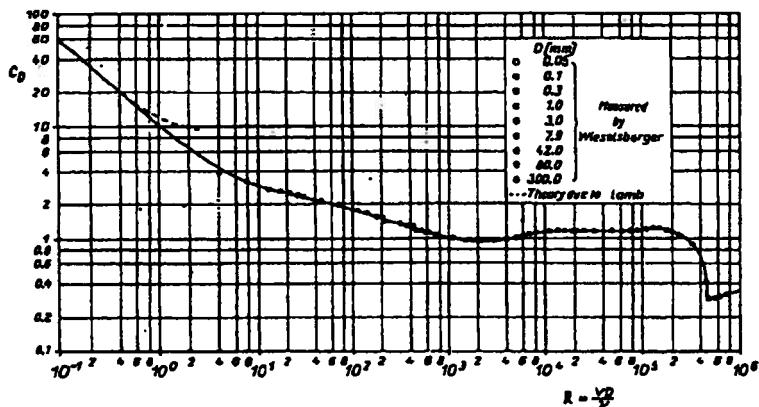
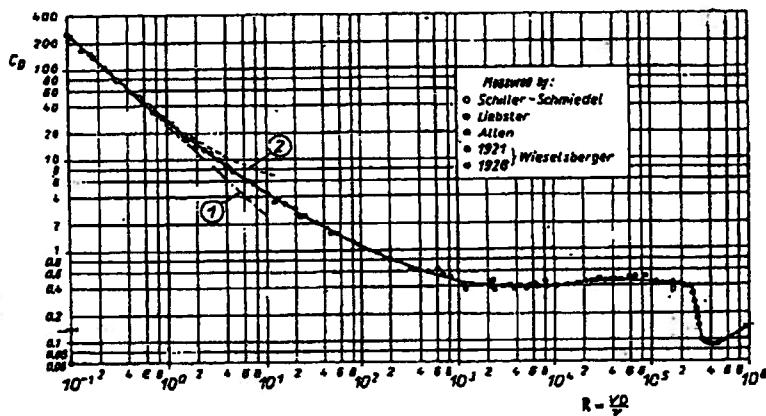


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number



Curve (1), Stokes' theory:  
 $C_D = 24/R$

Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number  
Curve (1): Stokes' theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.18)

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates ( $x, y, z$ )	Cylindrical Coordinates ( $r, \theta, z$ )
$\tau_{xx} = \mu \left[ 2 \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot v) \right]$	$\tau_{rr} = \mu \left[ 2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot v) \right]$
$\tau_{yy} = \mu \left[ 2 \frac{\partial v_y}{\partial y} - \frac{2}{3} (\nabla \cdot v) \right]$	$\tau_{\theta\theta} = \mu \left[ 2 \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) - \frac{2}{3} (\nabla \cdot v) \right]$
$\tau_{zz} = \mu \left[ 2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot v) \right]$	$\tau_{zz} = \mu \left[ 2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot v) \right]$
$\tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$
$\tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu \left[ \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$
$\tau_{zx} = \tau_{xz} = \mu \left[ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right]$	$\tau_{rz} = \tau_{zr} = \mu \left[ \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$
$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot v) = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

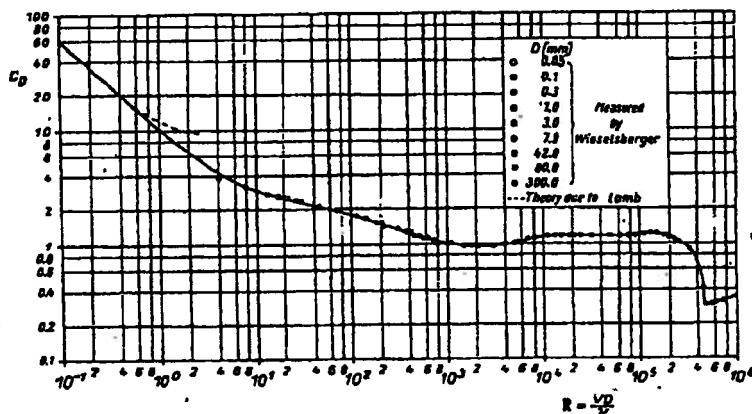
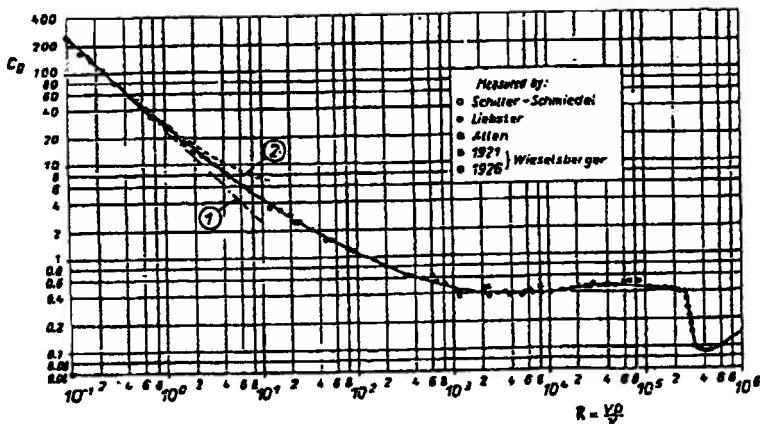


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number



Curve (1), Stokes' theory:  
 $C_D = 24/R$

**Table 5.2 Dimensionless Groups in Fluid Mechanics**

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$Re = \frac{\rho UL}{\mu}$	Inertia Viscosity	Always
Mach number	$Ma = \frac{U}{a}$	Flow speed Sound speed	Compressible flow
Froude number	$Fr = \frac{U^2}{gL}$	Inertia Gravity	Free-surface flow
Weber number	$We = \frac{\rho U^2 L}{\gamma}$	Inertia Surface tension	Free-surface flow
Rossby number	$Ro = \frac{U}{\Omega_{\text{earth}} L}$	Flow velocity Coriolis effect	Geophysical flows
Cavitation number (Euler number)	$Ca = \frac{p - p_{\text{v}}}{\rho U^2}$	Pressure Inertia	Cavitation
Prandtl number	$Pr = \frac{\mu c_p}{k}$	Dissipation Conduction	Heat convection
Eckert number	$Ec = \frac{U^2}{c_p T_0}$	Kinetic energy Enthalpy	Dissipation
Specific-heat ratio	$k = \frac{c_p}{c_v}$	Enthalpy Internal energy	Compressible flow
Strouhal number	$St = \frac{\omega L}{U}$	Oscillation Mean speed	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	Wall roughness Body length	Turbulent, rough walls
Grashof number	$Gr = \frac{\beta \Delta T g L^3 \rho^3}{\mu^3}$	Buoyancy Viscosity	Natural convection
Rayleigh number	$Ra = \frac{\beta \Delta T g L^3 \rho c_p}{\mu k}$	Buoyancy Viscosity	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	Wall temperature Stream temperature	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_{\infty}}{\frac{1}{2} \rho U^2}$	Static pressure Dynamic force	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2} \rho U^2 A}$	Lift force Dynamic force	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2} \rho U^2 A}$	Drag force Dynamic force	Aerodynamics, hydrodynamics
Friction factor	$f = \frac{h_f}{(V^2/2g)(L/d)}$	Friction head loss Velocity head	Pipe flow
Skin friction coefficient	$c_f = \frac{\tau_{\text{wall}}}{\rho V^2/2}$	Wall shear stress Dynamic pressure	Boundary layer flow

**Kinematic Viscosity of Glycerin-Water Mixture.** Mass % indicates percentage of the mixture mass that Glycerin makes up. For example, 0.5% corresponds to a mixture of 99.5%-water and 0.5%-Glycerin.

**Mass %      Kinematic Viscosity**  
**m<sup>2</sup>/s**

0.5	1.01181E-06
1	1.02149E-06
2	1.04507E-06
3	1.06855E-06
4	1.09192E-06
5	1.11617E-06
6	1.14328E-06
7	1.17114E-06
8	1.19996E-06
9	1.23248E-06
10	1.26383E-06
12	1.33015E-06
14	1.40142E-06
16	1.47973E-06
18	1.56595E-06
20	1.66077E-06
24	1.8824E-06
28	2.1371E-06
32	2.44847E-06
36	2.83928E-06
40	3.32575E-06
44	4.00559E-06
48	4.83304E-06
52	5.89494E-06
56	7.3115E-06
60	9.26366E-06
64	1.17298E-05
68	1.57014E-05
72	2.32808E-05
76	3.38769E-05
80	4.95856E-05
84	8.91749E-05
88	0.000119924
92	0.000309954
96	0.000623967
100	