Ph.D. Qualifying Exam

Fluid Mechanics

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Directions:

• Closed book, but one sheet (letter sized, front and back) of your own notes with equations permitted
• Some basic equations (if needed) are provided on an attached information sheet
• Answer all questions
• All questions have the same weighting

Time: 3.0 hours
Problem 1
The following diagram shows a Couette flow between two plates where the bottom wall is stationary and the top is moving at constant velocity. \( u \) is the velocity of the flow, which is a linear function of \( y \). The fluid can be assumed to be incompressible.

\[ u(y) \]

a) Can this flow considered to be a rotational flow (justify your answer mathematically)?
b) Find the volume flow \( (Q) \) between the two walls?
c) What is the ratio of volume flow \( (Q) \) between the upper and lower half of the flow?
**Problem 2**
Consider the following converging-diverging nozzle where the flow of air goes from left to right. The air accelerates to supersonic speeds as it exits from point 2. Conditions of the air at point 1 and 2 are as follows. Please answer the following questions.

**Point 1**: $P_1=284 \text{ kPa}$, $T_1=665 \text{ K}$, $V_1=517 \text{ m/s}$

**Point 2**: $P_2=8 \text{ kPa}$, $T_2=240 \text{ K}$

Some constants for ideal gas assumption
$k=1.4$, $R=287 \text{ m}^2/(\text{s}^2 \cdot \text{K})$

throat diameter at 1 is 0.01m in diameter
exit throat diameter is 0.025m in diameter

a) What is the mass flow in kg/s?
b) What is the Mach number at throat of the nozzle?
c) What is the Mach number at the exit of the nozzle?
Problem 3.

The gap between a plane moving wall and a stationary wall of height $H$ is filled with a liquid of density $\rho$. A seal prevents leakage of liquid past the sliding joint. The gap width $b$ is small relative to $H$ and so if one assumes inviscid flow, the velocity profile $w$ at any $z$ location is uniform and parallel.

If $\rho$ and the speed of the wall $V$ are constant, $g$ acts vertically downwards, and the pressure at elevation $H$ is atmospheric, find an expression for the static pressure $p(z)$ in fluid in the gap, at any elevation $z$. 
Problem 4.

The flight characteristics of a golf ball are to be tested using a model in a wind tunnel. The drag force $F_D$ on the golf ball is known to depend on its diameter $D$, its velocity $V$, its angular velocity $\omega$, the depth of dimples $d$, and the air density $\rho$ and viscosity $\mu$.

(i) Find the dimensionless parameters of this problem and express the functional dependence between them.

(ii) A golfer can hit a golf ball at $V = 240$ ft/sec and $\omega = 9000$ rpm. If these conditions are to be modeled in a wind tunnel with a velocity of 80 ft/sec, what diameter model golf ball should be used?

(iii) At what angular velocity should the model golf ball rotate?

(iv) How would this analysis change if the real golf ball could be hit at 750 ft/sec?

The diameter of a regulation golf ball is 1.68 in.
Problem 5.

A jet of water is directed against a vane as shown below. The water leaves the stationary 50 mm diameter nozzle with a uniform velocity of $V = 20$ m/s and flows along the inner surface of vane while maintaining the same cross-sectional area. If the surface of the vane at $B$ makes an angle of $150^\circ$ to the $x$-axis, find the force that must be applied to the vane to keep its speed constant at $U = 5$ m/sec.
Problem 6
You are looking down on a river following between two walls. You drop a cross shaped piece of wood onto the river (near the right side wall) and it floats down as shown in the figure. The velocity profile is given as

\[ u = u_{\text{max}} \left( 1 - \frac{y^2}{h^2} \right) \quad v = 0 \]

a) Derive a streamline function \( \psi \) for the fluid flow?
b) Will the piece of wood rotate as it travels down the river? If so then in which direction?
c) Will the piece of wood move closer to one of the walls as it travels down?