Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book, but one sheet (8.5” x 11”, front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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Problem 1:

The thrust from an airplane propeller is a function of the following variables:

\( V \) - speed of the airplane
\( D \) - diameter of the propeller
\( \rho \) - air density
\( \mu \) - air viscosity
\( c \) - speed of sound in air
\( \omega \) - rotational speed of the propeller

1. Find the dimensionless groups that characterize the thrust and give them names where appropriate.

2. Suppose a one-quarter scale model of the propeller is to be tested in an air flow and that effects of compressibility can be neglected. What values of \( V \) and \( \omega \) should be chosen for these tests?

3. Suppose effects of compressibility could not be ignored. Could full similarity still be achieved in an air flow for the one-quarter scale model? Explain why or why not.
Problem 2:

A Newtonian fluid flows steadily in the $x$-direction between two parallel plates a distance $h$ apart under the influence of a pressure gradient $dp/dx$, although no body forces are present. The flow is of unit depth, the lower plate at $y=0$ is stationary and the upper one at $y=h$ moves at velocity $U$ in the $x$ direction.

1. Give the simplest form of the $x$-momentum equation that describes this flow.

2. Find an expression for the velocity $u(y)$

3. Find an expression for the average velocity across this flow.

4. At what value of $dp/dx$ is the average velocity zero?
Problem 3:

A jet of liquid of density $\rho$ and area $A$ strikes a block and splits into two jets, as in the figure below. Assume the same uniform velocity $V$ for all three jets. The upper jet exits at an angle $\theta$ and area $\alpha A$. The lower jet is turned $90^\circ$ downward. Neglecting block and fluid weight as well as viscous effects:

a) Derive a formula for the forces $(F_x, F_y)$ required to support the block.

b) Show that $F_y = 0$ only if $\alpha \geq 0.5$.

c) Find the values of $\alpha$ and $\theta$ for which both $F_x$ and $F_y$ are zero. What is the block shape corresponding to these values?
Problem 4:

The inviscid, incompressible flow in the vicinity of a stagnation point (see Figure below) is approximated by \( u = -10x \) and \( v = 10y \). If the pressure at the origin is \( p_0 \), find an expression for the pressure neglecting gravity effects:

a) Along the negative x-axis
b) Along the positive y-axis
c) Would the procedure for obtaining the pressure in parts a and b above be different if the flow was viscous? If so, then describe the procedure for viscous flow (but don’t actually solve for the pressure).
Integral mass conservation equation:

\[ 0 = \frac{\partial}{\partial t} \int_C \rho d\mathbf{a} + \int_{CS} \rho \mathbf{V} \cdot d\mathbf{A} \]

Integral momentum equation:

\[ \mathbf{F} = \mathbf{F}_r + \mathbf{F}_n = \frac{\partial}{\partial t} \int_C \mathbf{V} \rho d\mathbf{a} + \int_{CS} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \]

Integral angular momentum equation:

\[ \sum (\mathbf{r} \times \mathbf{F}) = \frac{\partial}{\partial t} \int_C (\mathbf{r} \times \mathbf{V}) \rho d\mathbf{a} + \int_{CS} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A} \]

The Equation of Continuity

Rectangular Coordinates \((x, y, z)\):

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0 \]

Cylindrical Coordinates \((r, \theta, z)\):

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho u_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0 \]

Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity \((\mu)\)

Rectangular Coordinates \((x, y, z)\):

\[
\rho \left( \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \right) = \mu \left[ \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right] - \frac{\partial p}{\partial x} + \rho g_x
\]

\[
\rho \left( \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \right) = \mu \left[ \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right] - \frac{\partial p}{\partial y} + \rho g_y
\]

\[
\rho \left( \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \right) = \mu \left[ \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z
\]

Cylindrical Coordinates \((r, \theta, z)\):

\[
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_r v_r}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} \right) = \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_r) \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right] - \frac{\partial p}{\partial r} + \rho g_r
\]

\[
\rho \left( \frac{\partial v_\theta}{\partial t} + u_r \frac{\partial v_\theta}{\partial r} + \frac{u_r v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + u_z \frac{\partial v_\theta}{\partial z} \right) = \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] - \frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta
\]

\[
\rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_r v_z}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r}{r} \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z
\]
### Table C-3 Components of the Stress Tensor for Newtonian Fluids

<table>
<thead>
<tr>
<th>Rectangular Coordinates</th>
<th>Cylindrical Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y, z)$</td>
<td>$(r, \theta, z)$</td>
</tr>
<tr>
<td>$\tau_{xx} = \mu \left[ \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$</td>
<td>$\tau_{rr} = \mu \left[ \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$</td>
</tr>
<tr>
<td>$\tau_{yy} = \mu \left[ \frac{\partial v_y}{\partial y} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$</td>
<td>$\tau_{\theta\theta} = \mu \left[ \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right] - \frac{2}{3} (\nabla \cdot \mathbf{v})$</td>
</tr>
<tr>
<td>$\tau_{zz} = \mu \left[ \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$</td>
<td>$\tau_{z\theta} = \mu \left[ \frac{\partial v_z}{\partial \theta} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$</td>
</tr>
<tr>
<td>$\tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right]$</td>
<td>$\tau_{r\theta} = \tau_{\theta r} = \mu \left[ \frac{\partial v_r}{\partial \theta} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right]$</td>
</tr>
<tr>
<td>$\tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right]$</td>
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<td>$\tau_{zx} = \tau_{xz} = \mu \left[ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right]$</td>
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</tr>
</tbody>
</table>

$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number

![Drag coefficient for circular cylinders](image)

Curve (1), Stokes' theory: $C_D = \frac{24}{R}$

Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number

![Drag coefficient for spheres](image)

Curve (1): Stoke's theory, eqn. (6.10); curve (3): Oseen's theory, eqn. (6.13)