Thermodynamics
Ph.D. Qualifying Exam

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Directions: Work all four problems. Problems are equally weighted. Students are allowed one book, calculator, and five sheets of notes
1- A 30 liter electrical radiator containing heating oil is placed in a 5m x 4m x 2.5 m room. Both the room and the oil in the radiator are initially at 10 °C. The radiator with a rating of 1.8 kW is now turned on. At the same time, heat is lost from the room at an average rate of 0.35 kJ/s. After some time, the average temperature in the room is measured to be 20 °C for the air in the room and 50 °C for the oil in the radiator. Taking the density and the specific heat of the oil to be 950 kg/m3 and 2.2kJ/kg°C, respectively, determine how long the heater is kept on. Assume the room is well sealed so that there are no air leaks and assume constant specific heats at room temperature (note the properties of air at room temperature are $R = 0.287 \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$ and $c_v = 0.718 \text{kJ/kg-K}$).

We take the air in the room and the oil in the radiator to be the system. This is a closed system since no mass crosses the system boundary. The energy balance for this stationary constant-volume closed system can be expressed as

\[
\frac{E_{\text{in}} - E_{\text{out}}}{\text{Net energy transfer by heat, work, and mass}} = \frac{\Delta E_{\text{system}}}{\text{Change in internal, kinetic, potential, etc. energies}}
\]

\[
(\dot{W}_{\text{in}} - \dot{Q}_{\text{out}})\Delta t = \Delta U_{\text{air}} + \Delta U_{\text{oil}}
\]

\[
\equiv [mc_v(T_2 - T_1)]_{\text{air}} + [mc_p(T_2 - T_1)]_{\text{oil}} \quad \text{(since KE = PE = 0)}
\]

The mass of air and oil are

\[
m_{\text{air}} = \frac{pV_{\text{air}}}{RT_1} = \frac{(100 \text{ kPa})(50 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(10 + 273K)} = 61.56 \text{ kg}
\]

\[
m_{\text{oil}} = \rho_{\text{oil}}V_{\text{oil}} = \left(950 \frac{\text{kg}}{\text{m}^3}\right)(0.03 \text{ m}^3) = 28.5 \text{ kg}
\]

Substituting,

\[
\left(1.8 - 0.35 \frac{\text{kJ}}{\text{s}}\right)\Delta t = (61.56 \text{ kg}) \left(0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{°C}}\right)(20 - 10)\text{°C} + (28.5 \text{ kg})(2.2\text{kJ/kg} \cdot \text{°C})(50 - 10)\text{°C}
\]

\[
\Delta t = 2034s = 33.9\text{min}
\]
2- A coal burning steam power plant produces a net power of 300 MW with an overall thermal efficiency of 32%. The actual gravimetric air-fuel ratio in the furnace is calculated to be 12 kg air/kg fuel. The heating value of the coal is 28,000 kJ/kg. Determine:

a- The amount of coal consumed during a 24 hour period
b- The rate of air flowing through the furnace

(a) The rate and the amount of heat inputs to the power plant are

\[
\dot{Q}_{in} = \frac{\dot{W}_{net, out}}{\eta_{th}} = \frac{300 \text{ MW}}{0.32} = 937.5 \text{ MW}
\]

\[
Q_{in} = \dot{Q}_{in} \Delta t = (937.5 \text{ MJ/s})(24 \times 3600 \text{ s}) = 8.1 \times 10^7 \text{ MJ}
\]

The amount and rate of coal consumed during this period are

\[
m_{\text{coal}} = \frac{Q_{in}}{q_{HV}} = \frac{8.1 \times 10^7 \text{ MJ}}{28 \text{ MJ/kg}} = 2.893 \times 10^6 \text{ kg}
\]

\[
\dot{m}_{\text{coal}} = \frac{m_{\text{coal}}}{\Delta t} = \frac{2.893 \times 10^6 \text{ kg}}{24 \times 3600 \text{ s}} = 33.48 \text{ kg/s}
\]

(b) Noting that the air-fuel ratio is 12, the rate of air flowing through the furnace is

\[
\dot{m}_{\text{air}} = (AF)\dot{m}_{\text{coal}} = (12 \text{ kg air/kg fuel})(33.48 \text{ kg/s}) = 401.8 \text{ kg/s}
\]
3- A turbofan engine operating on an aircraft flying at 200 m/s at an altitude where the air is at 50 kPa and -20°C is to produce 50,000 N of thrust. The inlet diameter of this engine is 2.5m, the compressor pressure ratio is 12, and the mass flow rate ratio is 8. Determine (a) the mass flow rate through the fan, (b) the velocity at the fan exit, and (c) the air temperature at the fan outlet needed to produce this thrust. Assume ideal operation for all components and constant specific heats at room temperature (note the properties of air at room temperature are $R = 0.287 \text{ kPa-m}^3/\text{kg-K}$, $c_p = 1.005 \text{ kJ/kg-K}$ and $k = 1.4$).

The total mass flow rate is

$$\dot{m} = \frac{AV_1}{\nu_1} = \frac{\pi D^2 V_1}{4 \nu_1} = \frac{\pi(2.5 \text{ m})^2}{4 \cdot 1.452 \text{ m}^3/\text{kg}} = 676.1 \text{ kg/s}$$

Now, $\dot{m}_e = \frac{\dot{m}}{8} = \frac{676.1 \text{ kg/s}}{8} = 84.51 \text{ kg/s}$

(a) The mass flow rate through the fan is

$$\dot{m}_f = \dot{m} - \dot{m}_e = 676.1 - 84.51 = 591.6 \text{ kg/s}$$

(b) In order to produce the specified thrust force, the velocity at the fan exit will be

$$F = \dot{m}_f (V_{\text{exit}} - V_{\text{inlet}})$$

$$V_{\text{exit}} = V_{\text{inlet}} + \frac{F}{\dot{m}_f} = (200 \text{ m/s}) + \frac{50,000 \text{ N}}{591.6 \text{ kg/s}} \left( \frac{1 \text{ kg-m/s}^2}{1 \text{ N}} \right) = 284.5 \text{ m/s}$$

(c) An energy balance on the stream passing through the fan gives the air temperature at the fan outlet

$$c_p(T_4 - T_5) = \frac{V_{\text{exit}}^2 - V_{\text{inlet}}^2}{2}$$

$$T_5 = T_4 - \frac{V_{\text{exit}}^2 - V_{\text{inlet}}^2}{2c_p}$$

$$= 253 \text{ K} - \frac{(284.5 \text{ m/s})^2 - (200 \text{ m/s})^2}{2(1.005 \text{ kJ/kg-K}) \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)}$$

$$= 232.6 \text{ K}$$
4. Decane is burned with 120% theoretical air is a constant pressure process at 1 atm and the products are cooled to 25°C. Calculate the number of kilograms of liquid water in the products per kg of decane burned. $\text{MW}_{H_2O} = 18.015$, $\text{MW}_{C_{10}H_{22}} = 142.26$.

The equation for stoichiometric decane combustion is:

$$C_{10}H_{22} + 15.5O_2 + 15.5(3.76)N_2 \rightarrow 10CO_2 + 11H_2O + 58.28N_2$$

For 120% Theoretical air:

$$\frac{y_{H_2O}}{n_{H_2O}} = \frac{P_a}{P} = \frac{3.169}{101} = \frac{n_{H_2O\text{max}}}{n_{H_2O\text{max}} + (10 + 310 + 6994)}$$

$$101n_{H_2O} = 3.169(n_{H_2O} + 83.04)$$

$$n_{H_2O} = 2.68 \times \text{number of moles of H}_2\text{O vapor}$$

$$n_{H_2O} = 11 - 2.68 = 8.32$$

$$\frac{w_{H_2O\text{vap}}}{m_{C_{10}H_{22}}} = \frac{8.32(18.015)}{142.28} = \frac{1.06 \text{ kg H}_2\text{O}}{\text{kg C}_{10}\text{H}_{22}}$$

5. Elon Musk has advertised his new battery powered Class 8 truck to be a breakthrough. The gross vehicle weight of a Class 8 truck is 80,000 lbs, the maximum weight allowed by law to operate on a public highway. A modern diesel powered class 8 truck can haul between 40000+ pounds of payload and with an
engine output of 200kW can travel at an average speed of 60 MPH. Assume the electric truck and trailer weight of 40000 lbs empty, not including batteries. Assume the battery specific weight/energy density is the same as the new Chevrolet Bolt battery (about 60 kW-hr for a 960 lb battery) and that a travel of 550 miles is required (Detroit to Washington DC). Without stopping to recharge, how much actual payload weight can the driver haul with the battery powered truck over the journey? How does this compare to the actual payload of a Ford pickup truck for the same trip? As a point of reference, a 2018 Ford F250, 2 wheel drive has a payload capacity of 16000 lbs. The load capacity of 16000 lbs. accounts for the aluminum trailer weight but not for the fuel used in the journey. Diesel fuel weighs about 6.9 lbs/gallon and the fuel economy of the Ford pickup would be about 10 mpg with a 16000 payload.

\[
\begin{align*}
\text{550 miles} & = \frac{9.17 \text{ hrs}}{60 \text{ miles/hr}} \\
\text{Energy required} & = \frac{9.17 \text{ hrs} \times 200 \text{ kW}}{1833 \text{ kWh/km}} \\
\text{Battery weight reqd} & = \frac{1833 \text{ kWh}}{60 \text{ kWh/km}} \times 1960 \text{ lbs} = 29333 \text{ lbs} \\
\text{Payload} & = 80000 - 40000 - 29333 = 10666 \text{ lbs}
\end{align*}
\]

If F250 diesel gets 10 mpg pulling a trailer with a 16000 lb payload:

Ford F250 fuel reqd = 550 mi / 10 mi/gal \times 6.9 lb/gal = 379

\[
\text{Payload} = 16000 - 379 = 15621 \text{ lbs}
\]

F250 \rightarrow 15,621
Tesla Semi \rightarrow 10,667