Student ID _____

# Fluid Mechanics Qualifying Exam
Department of Mechanical Engineering

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Closed book, but formula sheets are provided.

All questions have the same weight.

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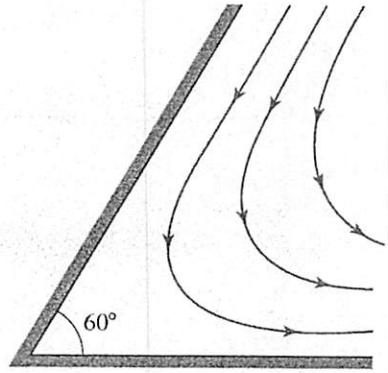
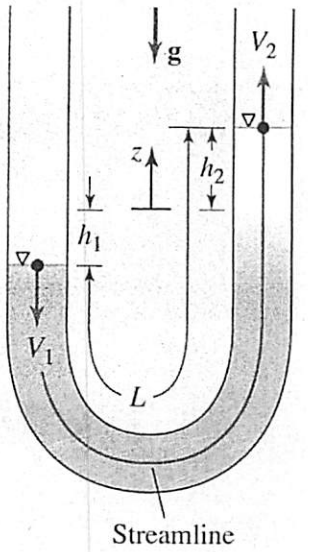
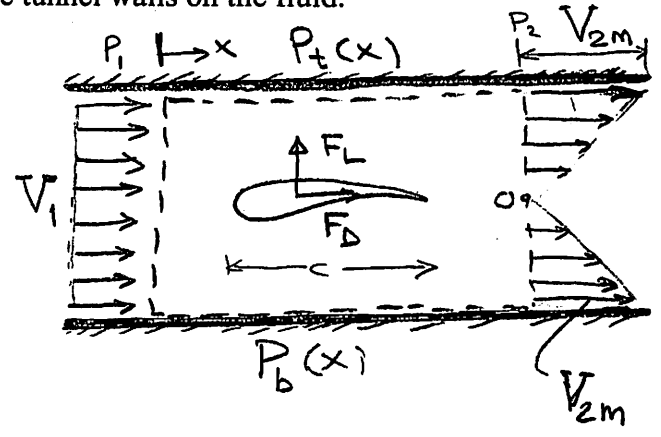
**Problem 1.** The flow illustrated in the figure is given by $u=A(x^2-y^2)$, $v=-2Axy$, $w=0$, where $u$, $v$ *and* $w$ are velocity components and A is a constant. Check if this is an irrotational flow. If so, find the velocity potential and demonstrate that it satisfies Laplace's equation $\nabla^2\phi=0$. Also, check to see if the continuity equation is satisfied.



**Problem 2.** A liquid column of total length L oscillates in a U-tube as shown in the figure. Assuming frictionless flow, find an expression predicting the frequency of oscillation.



**Problem 3.** A two dimensional airfoil is placed in a constant area wind tunnel with height h and width W. The incoming flow is uniform with contact velocity $V_1$ and pressure $P_1$. The velocity field downstream of the airfoil, $V_2$ is not uniform but may be assumed to linearly increase from zero at the centerline to $V_{2m}$ close to wall. The flow pressure at this downstream location, $P_2$ is lower than $P_1$. The pressure at top and bottom surfaces of the channel are $P_t(x)$ and $P_b(x)$, respectively. Compute the lift and drag forces generated by the airfoil as a function of $V_1$ , $P_1$ , $P_2$ , $P_t(x)$ , $P_b(x)$ , h , W. What are the lift and drag coefficients? (hint: lift and drag coefficients are lift and drag forces normalized by the dynamic pressure ). Assume the flow streamlines at stations 1 and 2 to be parallel and neglect the viscous forces exerted by the tunnel walls on the fluid.



**Problem 4.** Simplify the compressible Navier-Stokes equations (attached continuity, momentum and energy equations) for a two-dimensional, steady, incompressible flow in the x-y plane. Ignore gravity. Can you simplify these equations further for a flat plat laminar boundary layer (FPLBL) by scaling? If yes, show how. Write the boundary conditions for the FPLBL flow. How do you solve the simplified equations for a FPLBL? (you do not need to solve them just state how).

# Useful Equations

Integral mass conservation equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Integral momentum equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A}$$

## The Equation of Continuity

Rectangular Coordinates $(x, y, z)$:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates $(r, \theta, z)$:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

## Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity ($\mu$)

Rectangular Coordinates $(x, y, z)$:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates $(r, \theta, z)$:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

## Vorticity Equation in Rectangular Coordinates :

$$\vec{\Omega} = \Omega_x \vec{i} + \Omega_y \vec{j} + \Omega_z \vec{k}$$

Vorticity Vector

$$\Omega_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}$$

$$\Omega_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}$$

$$\Omega_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}$$

## Bernoulli Equation For Compressible Unsteady Flows :

$$\rho\frac{\partial V}{\partial t}ds + dp + \frac{1}{2}\rho d(V^2) + \rho g\, dz = 0 \quad \text{(along a streamline)}$$

$V = |\vec{V}|$ $\quad p \equiv$ pressure $\quad s \equiv$ coordinate along streamline