Code Number: _________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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<u>Directions:</u>

Work all four problems.
The problems carry equal weight.
You may use two books for reference.

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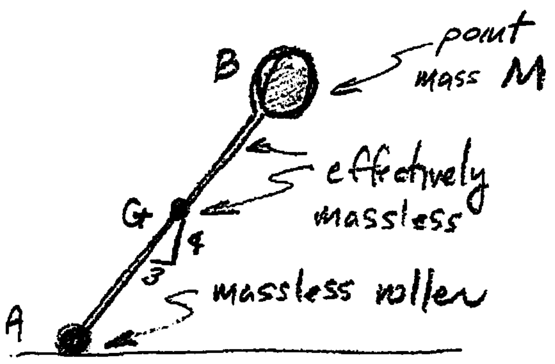
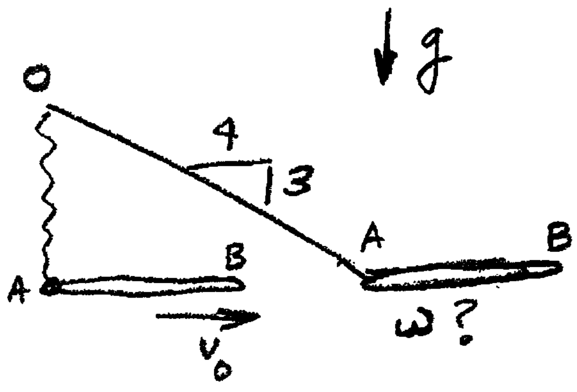
1. The mass M is at the end B of a rigid rod AB of negligible mass. At the other end A is a massless roller. The system is released from rest in the configuration as shown and falls under the action of gravity. At the instant after release, find the acceleration $\mathbf{a}_G$ of the midpoint G of the rod (in terms of the gravitational acceleration g). Express your answer as a vector in Cartesian (rectangular) coordinates.



2. The uniform rod AB of mass M and length L is attached by an inextensible chord at end A. The other end of the chord is attached to a fixed point O. Initially the chord is slack and the rod is translating (no spin) to the right with speed v_0. When the chord becomes taut, it is aligned with the hypotenuse of a 3-4-5 right triangle as shown. Find the angular velocity ω <u>of the rod</u> at the instant after the chord becomes taut.



3. (five parts) Consider a single degree of freedom model of a resonator that is driven by base excitation through a damping element, as shown in Figure 3.1 below. The inertial displacement of m is denoted as $x(t)$, and the base excitation through the dashpot c on the right side is from a prescribed displacement $y(t)$. Using standard notation, we let $h(t)$ be the unit impulse response and $g(t)$ be the unit step response of the system, where here the impulse and step refer to input *forces.*

(a) Show that the equation of motion is given by $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 2\zeta\omega_n\dot{y}$, where $\zeta = c/(2m\omega_n)$ is the damping ratio and $\omega_n = \sqrt{k/m}$ is the undamped natural frequency.

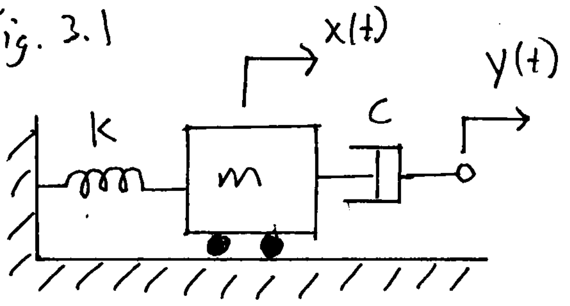
(b) Consider a step *displacement* to the end of the dashpot, that is, $y(t) = Y_s u(t)$, where $u(t)$ is the unit step function and Y_s is the amplitude of the step. Express the response $x(t)$ to this input in terms of the system parameters, Y_s, and either $h(t)$ or $g(t)$.

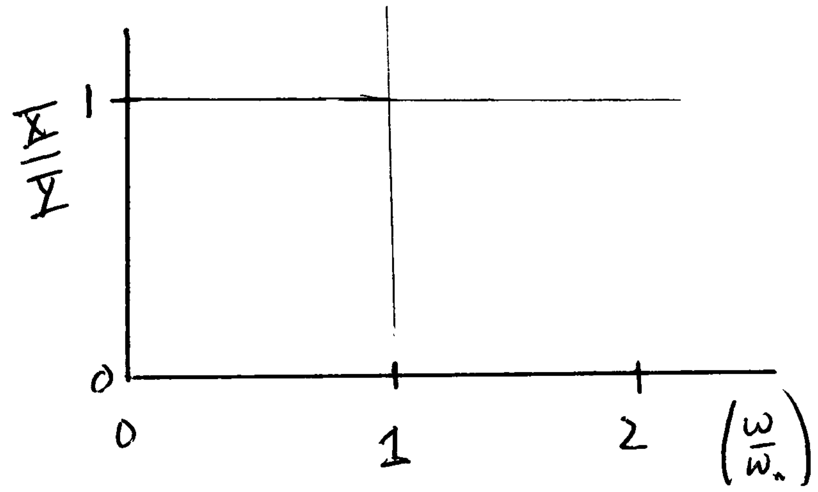
(c) Consider the input to be harmonic and given by $y(t) = Y\sin(\omega t)$. Determine the amplitude X and phase ϕ of the resulting steady-state response when expressed in the form $x_{ss} = X\cos(\omega t - \phi)$. This is generally easiest to do using complex variables.

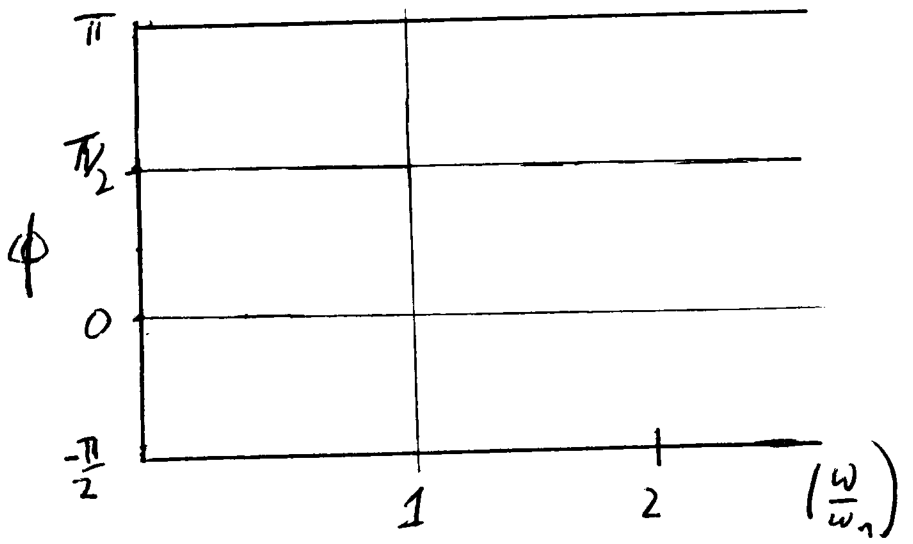
(d) Sketch X and ϕ versus ω/ω_n on the graphs provided below; show the features of the graphs as clearly as possible, and clearly label specific values at $\omega = 0, 1, \rightarrow \infty$.

(e) This system does not experience a large amplitude resonance at $\omega = 1$, even for light damping, $\zeta << 1$; show this mathematically AND give a physical reason for this fact.

Fig. 3.1







4. (eight parts) Consider the three degree-of-freedom (DOF) system shown in Figure 4.1 below, which is described by lumped mass and spring elements. The displacements of masses m_i are given by x_i, and the forces applied to the m_i's are represented by f_i, for $i = 1, 2, 3$. The displacement and force column vectors are defined as $\mathbf{x} = (x_1 \ x_2 \ x_3)^{\mathrm{T}}$ and $\mathbf{f} = (f_1 \ f_2 \ f_3)^{\mathrm{T}}$, respectively, where $(\cdot)^{\mathrm{T}}$ denotes the transpose of $(\cdot)$. The inertia and stiffness matrices, in some appropriate units, are given by

$$\mathbf{M} = \begin{pmatrix} m & 0 & 0 \\ 0 & \alpha\, m & 0 \\ 0 & 0 & m \end{pmatrix} \quad \text{and } \mathbf{K} = \begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{pmatrix},$$

so that the equations of motion have the form $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$.

NOTE 1: You can solve these questions without normalizing the modal vectors.

NOTE 2: Some of the following questions can be answered by physical reasoning with very little calculations, and you are welcome, in fact, encouraged, to do the problems this way, provided you give a good explanation for your approach.

(a) Show that the system natural frequencies are $\omega_1 = 0$, $\omega_2 = \sqrt{\frac{k}{m}}$, and $\omega_3 = \sqrt{\frac{(2+\alpha)}{\alpha}\frac{k}{m}}$.

(b) Prove that $\omega_1 = 0$ must follow from the fact that $\det(\mathbf{K}) = 0$.

(c) Determine the system mode shapes in the form $\tilde{\mathbf{u}}_j = (1 \ u_2 \ u_3)^{\mathrm{T}}$ for $j = 1, 2, 3$.

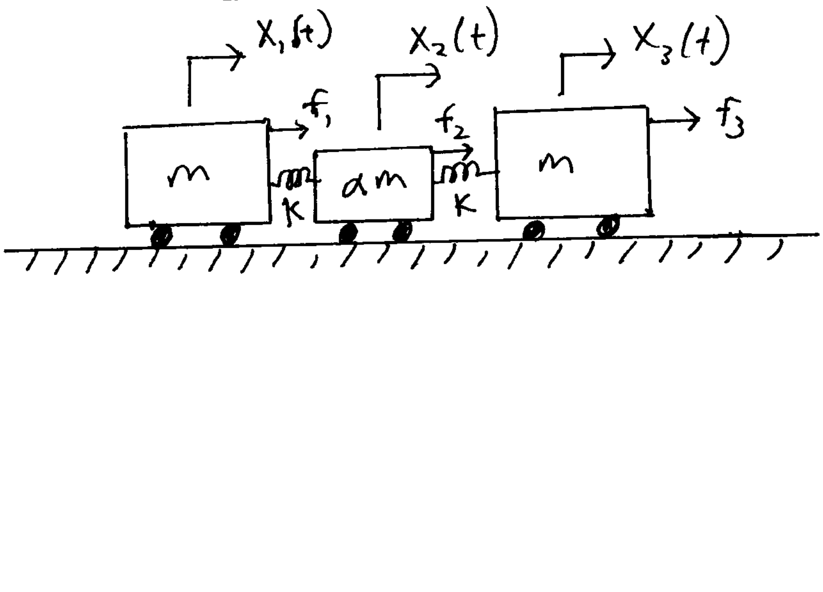
(d) The mode shape corresponding to $\omega_1 = 0$ is a rigid body mode; prove that this mode shape is an eigenvector of $\mathbf{K}$.

(e) Show that for the mode shapes corresponding to ω_2 and ω_3, the system center of mass is stationary, AND give a physical reason for this.

(f) Consider the situation in which the system is sitting at rest and all the masses are simultaneously struck by equal step inputs of magnitude F_o, that is, $\mathbf{f}(t) = F_o\, u(t)\ (1 \ 1 \ 1)^{\mathrm{T}}$ where $u(t)$ is the unit step function. Determine the frequency content of the resulting motion, that is, state the frequencies that are present in the response. Be sure to include ALL possible natural frequencies present in your answer.

(g) Consider the case for which harmonic forcing is applied to $m_2 = \alpha m$ only, that is, the forcing is of the form $\mathbf{f}(t) = (0 \ \ f_o \sin(\omega t) \ \ 0)^{\mathrm{T}}$. Determine ALL values of the excitation frequency ω at which resonance will occur.

(h) Explain why it would be a bad idea to place a sensor, such as an accelerometer, on m_2 if one were carrying out experiments to determine the vibration characteristics of this system.



$m_1 = m$
$m_2 = \alpha m$
$m_3 = m$