Student Code Number:____________

Ph.D. Qualifying Exam:

Dynamics and Vibrations

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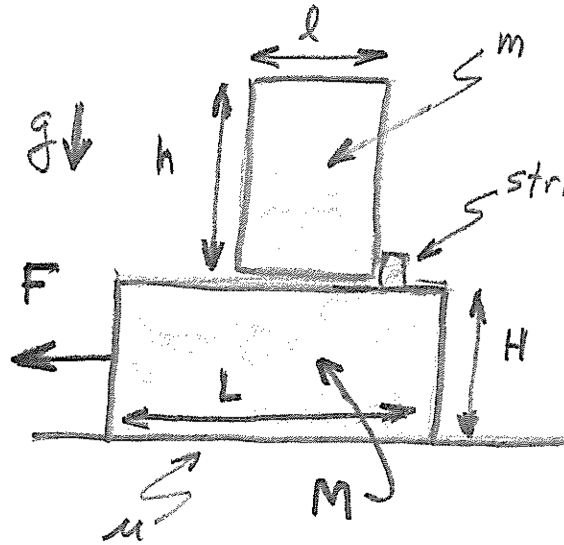
Directions:
Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two sheets of notes for reference.
No cell phones.

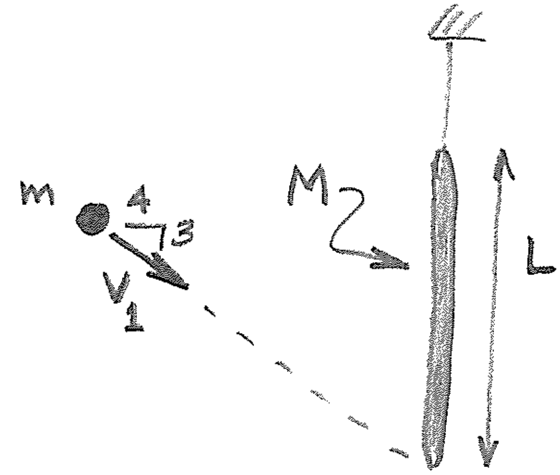
1. Blocks with uniform mass M and m are stacked as shown and initially at rest. The dimensions are L and H, and l and h, also as shown (lower case is the top block, upper case is the bottom block). A small strip is nailed to the lower block to prevent the top block from sliding when the bottom block is accelerated to the left. There is friction where the bottom block is in contact with the floor, and the coefficients of static and dynamic friction are the same and equal to $\mu>0$. The system is set in motion by the sudden application of a horizontal leftward force $\mathbf{F}$ to the bottom block.



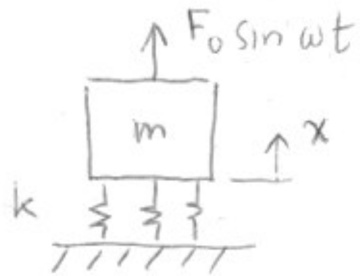
(a) Find the maximum horizontal force $\mathbf{F}= \mathbf{F}_{max}$ that can be applied without causing the upper block to tip. Express you answer for $\mathbf{F}_{max}$ in terms of m, l, h, M, L, H, μ and g (the gravity acceleration)..

(b) Suppose that the suddenly applied force is $\mathbf{F}= \beta\ \mathbf{F}_{max}$ with $\beta>1$. Find the angular acceleration α of the top block. Express you answer for α in terms of m, l, h, M, L, H, g, μ and β.

2. The uniform rod of mass M and length L is hanging vertically by a massless inextensible cable. A sticky particle of mass m is moving with speed v_1 as shown, causing it to strike the end of the rod and to stick there. Calculate the fractional loss in kinetic energy to the rod-particle system due to the collision. Your answer should be in terms of M, m, L and v_1.



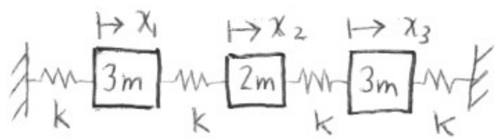
3. A machine mounted to the ground is represented as a vibration system with displacement x, as sketched. It has mass m = 10 kg, total stiffness k = 4000 N/m, negligible damping, is forced harmonically with the force $F(t) = F_0 \sin \omega t$. When forced at a frequency of $\omega = 10\sqrt{2}$ rad/sec, the response amplitude is X = 1 mm. It is required that the amplitude of the dynamic force transmitted (i.e. neglecting the static force of gravity) is limited to F_T = 1 N. The isolation system is to be designed first by adding mass, and second by changing the stiffness, such that the system achieves the desired transmitted force.
Determine
(a) The amplitude X when the transmitted force is reduced by adding mass.
(b) The amplitude X when the transmitted force is reduced by changing the stiffness.



4. The three-degree-of-freedom system sketched below has an equation of motion written as $\underline{M}\ddot{\underline{x}} + \underline{C}\dot{\underline{x}} + \underline{K}\underline{x} = \underline{F}(t)$.
The damping is proportional, such that $\underline{C} = \alpha\underline{M} + \beta\underline{K}$, and is associated with known modal damping factors $\zeta_1 = 0.05$ and $\zeta_2 = 0.1$ for the first two modes. The modal frequencies *squared* are $\omega_1^2 = 4$, $\omega_2^2 = 16$, and $\omega_3^2 = 24$, in rad^2/sec^2. The mass and stiffness matrices, the undamped *non-normalized modal matrix* $\underline{P}$, and forcing vector are given below:

$$\underline{M} = \begin{bmatrix} 3m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 3m \end{bmatrix},\ \underline{K} = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix},\ \underline{P} = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix},\ \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

$$\underline{F}(t) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} F\sin(\omega t)$$



Although scalar quantities m and k are not specified, the ratio is such that the modal frequencies given above are produced. Scalar F is also not specified.

(a) Find the damping factor ζ_3 of the third mode.

(b) Find the steady-state response of the *second mass* (i.e. find $x_2(t)$) when $\omega = 2$ rad/sec. You can write your answer in terms of unknown F, m and/or k as needed.