Student Code Number:____________________

Ph.D. Qualifying Exam: Dynamics and Vibrations

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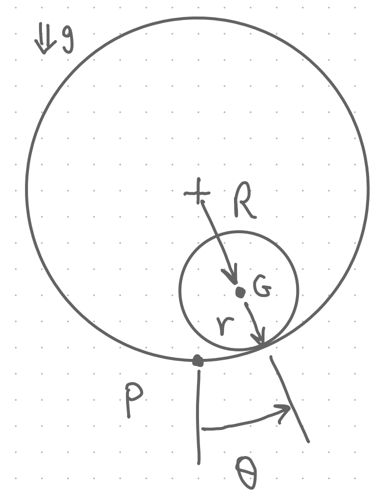
B. F. Feeny and F. A. Khasawneh

Directions:

- Work all four problems.
- Note that the problems are EVENLY WEIGHTED.
- You may use two books and two sheets of notes for reference.
- No cell phones or headphones.

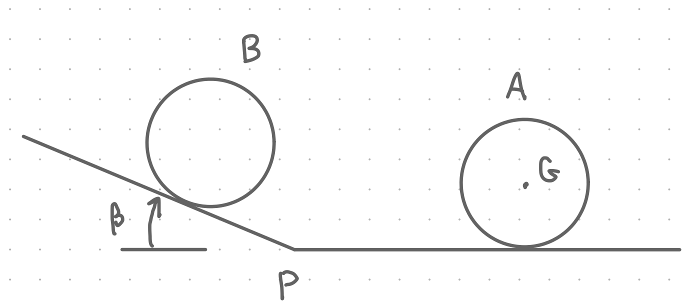
1. A uniform disk rolls within a larger cylinder that is fixed in space, as depicted. The rolling disk has radius r, and the larger cylinder has radius $r + R$, such that the center of mass of the disk is at a distance R from the center of the cylinder. The angle θ from the lowest position defines the position of the center of the disk within the cylinder. The mass moment of inertia of the disk about its center is given as $I_G = \frac{1}{2}mr^2$.



(a) If the disk is at $\theta = 0$ and the center of mass has a velocity v, and rolls without slip, determine the expression for v such that the maximum height of the disk is $\theta = \pi/2$.

(b) If the maximum angle that the disk can reach without slipping is $\theta_{\max}$, determine the static coefficient of friction.

2. A ball of radius r rolls along a horizontal surface, as depicted at point A, and then reaches an incline at point P, after which it rolls up the incline as depicted at point B. The incline is at the angle β, as shown in the figure. If the ball has angular velocity ω_0 on the horizontal surface immediately before reaching the incline at point P, and then meets the incline without bouncing or slipping, what is the angular velocity ω_1 on the incline immediately after reaching the incline at P? The mass moment of inertia of the ball about its center is given as $I_G = \frac{2}{5}mr^2$.

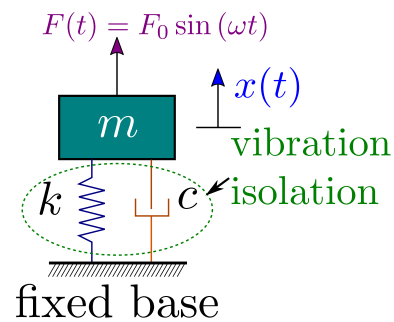


Problem 3. A small turbine, having a small unbalance that causes a harmonic force in the vertical direction as shown, weighs 800 N and operates at a speed of 600 rpm. A vibration isolation system is to be designed such that it satisfies two requirements: (1) It is desired to limit the response to a trasnmissibility of 2.5 as the turbine passes through resonance during start-up, and (2) 90% of the force is to be transmitted from the turbine to the support at the operating speed of the turbine. Answer the following questions:

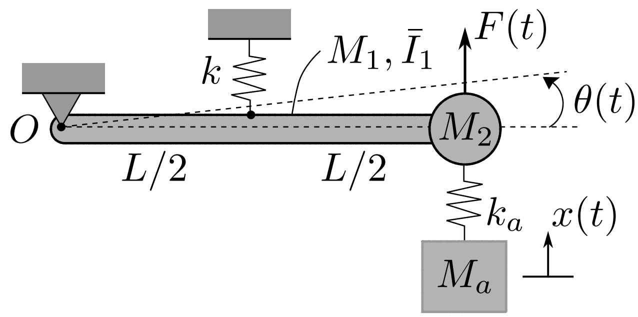
(a) Design a suitable isolator (find k and c) for the turbine that satisfies both requirements. (10 points)

(b) Independent of the answer of part (a), what is the magnitude of the steady state velocity of the vertical vibration of the turbine if we use an isolator with $k = 200$ KN/m, $c = 1800$ N.s/m, and the speed of the turbine is changed to 800 rpm while $F_0 = 2000$ N? (10 points)

(c) What is the magnitude of the force exerted by the turbine onto the floor under the conditions given in part (b)? Your answer should remain in symbolic form, i.e., do not plug in any numeric values. (5 points)



Problem 4. A bar of mass $M_1 = m$ and length L is vibrating due to external force $F(t)$. The bar carries a payload that can be approximated as point mass $M_2 = 3m$. The mass moment of inertia of the rod only without the payload about O is given by $I_O = \frac{1}{3}M_1L^2$. A vibration absorber of mass $M_a = 2m$ and stiffness $k_a = 2k$ is attached as shown to suppress the vibration.



(a) Assume small angle θ and define a variable $y = L\theta$. Show that the equations of motion of the system are written as (5 points)

$$\begin{bmatrix} 2m & 0 \\ 0 & \frac{10}{3}m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 2k & -2k \\ -2k & \frac{9}{4}k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \end{bmatrix}$$

(b) Suppose $F(t) = F_0 \sin(\omega t)$. **Derive** the expressions for the steady-state amplitudes $\begin{bmatrix} X \\ Y \end{bmatrix}$. For this part, your answer does not need to expand any determinant expressions. (10 points)

(c) **Derive** the tuning condition and use it to determine the numerical values of k and m such that the primary system's steady state response is $Y = 0$, and the vibration absorber's steady state amplitude is $X = 0.05$ m. Assume $\omega = 2$ rad/s and $F_0 = 1$ N. (10 points).