Student Code Number:____________

# Ph.D. Qualifying Exam:
# Dynamics and Vibrations

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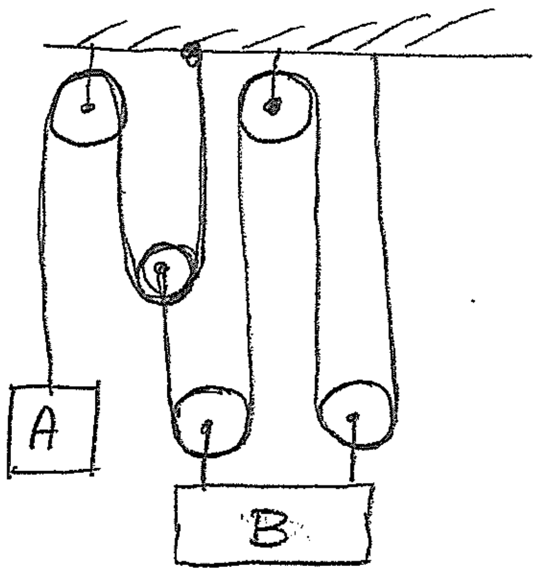
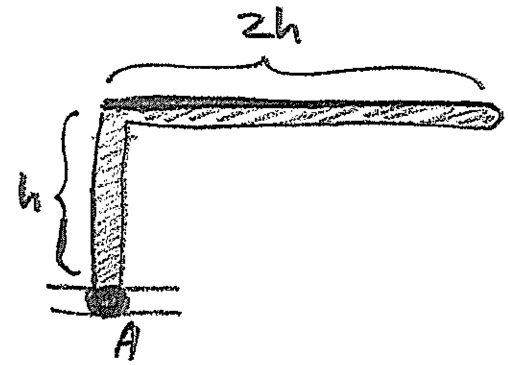
Directions:
Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.
No cell phones.

1. The system consists of two masses (A and B), two cables, and five pulleys. The pulleys are massless and are all well lubricated. The cables are also massless and inextensible. Under the action of gravity, it is observed that mass B is accelerating downward at one-half g (g is the gravitational acceleration 9.81 m/sec$^2$). If A has a mass of one kilogram, what is the mass of B?



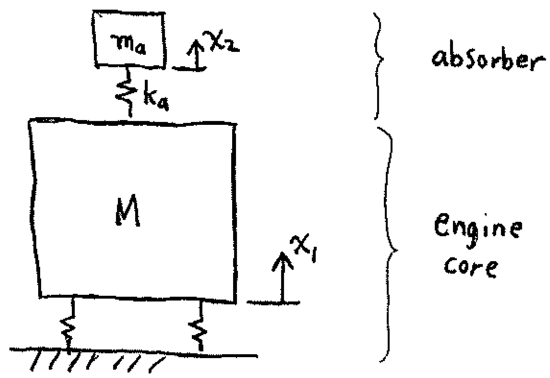
2. The rigid bar has two segments of length h and 2h as shown. The shorter segment is made of a lightweight material and can be regarded as massless. The longer segment is made of a heavier material with uniformly distributed mass over the segment and overall mass M. End A is free to slide in a horizontal track (massless roller). The system is released from rest in the configuration shown with gravity driven motion (no friction).




25% (A) Find the reaction at the roller and the angular acceleration of the bar at the instant of release.

75% (B) Find the reaction at the roller and the angular acceleration of the bar after the bar has rotated through 90 degrees ($\pi/2$ radians).

3. A large mass is externally forced with magnitude $F$ and frequency $\omega = 20$ rad/sec. For normal operation, a vibration absorber is to be attached, with the requirement that its response amplitude is 5 mm or less when tuned. First, a 10-kg test absorber ($m_{at} = 10$ kg) is attached as depicted below. At the given forcing frequency, the large mass is stationary ($X_1 = 0$), and the test absorber amplitude is $X_{2t} = 0.008$ m. What are the required mass and stiffness of the absorber to be used for normal operation. What is the value of the force magnitude, $F$?



4. A three DOF system has the mass and stiffness matrices, one *non-normalized* modal vector, and excitation vector, as

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix}, \quad K = \begin{bmatrix} 2k & -k & 0 \\ -k & 3k & -2k \\ 0 & -2k & 4k \end{bmatrix}, \quad \underline{u}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \underline{F} = \begin{pmatrix} F \\ 0 \\ 0 \end{pmatrix} \sin\omega t$$

(a) Find the second modal frequency

(b) Find the other modal vectors, as well.

(c) Suppose we have proportional damping. That is, suppose $M\ddot{\underline{x}} + C\dot{\underline{x}} + K\underline{x} = \underline{F}(t)$, and we have $C = \alpha M + \beta K$, with $\alpha = 0.05$ sec$^{-1}$ and $\beta = 0$ sec. The *normalized* modal matrix is defined as P = [$\underline{u}_1$, $\underline{u}_2$, $\underline{u}_3$], including the *normalized* $\underline{u}_2$ associated with the non-normalized vector given above. After letting $\underline{x} = P\underline{q}$, write out the differential equation for the second modal coordinate $q_2$.

(d) Suppose the mass (m = 2 kg), stiffness (k = 25 N/m), and forcing are such that the second modal coordinate has the differential equation

$$\ddot{q}_2 + 0.05\dot{q}_2 + 25q_2 = 5\sin\omega t$$

where the units in this equation are consistent with the formulation defined in part (c). If the excitation frequency is at the resonance of the second mode, i.e. $\omega = \omega_2$, and its modal damping is light, then *approximate* the amplitude $X_3$ of the steady-state response of the original coordinate $x_3$. (State your assumptions about resonance with small damping when you make this approximation.)