Student Code Number:_____

Ph.D. Qualifying Exam

Dynamics and Vibrations

August 2011

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<u>Directions:</u>   Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.
No cell phones.

1. Consider the uniform bar of mass m and length ℓ pivoted at one end (point O) by a frictionless bearing so that it moves in a vertical plane under the action of gravity. The angular position of the bar is measured by the angle θ shown. You are to consider the situation in which the bar is released from the horizontal position, $\theta = \pi/2$, with zero initial angular velocity.

(a) Determine the angular velocity $\dot{\theta}$ as a function of θ and the system parameters (m, ℓ, g) during the resulting motion.

(b) Determine the reaction force $\vec{R}_o$ acting on the bar at the support point O during the motion; express this as a function of θ and the system parameters, using unit vectors that you define and that are clearly marked on the diagram below.

(c) From the result of (b), or some other method of your choice, determine the minimum and maximum values of the magnitude of the reaction force, $R_o = ||\vec{R}_o||$, and the angles at which these occur.
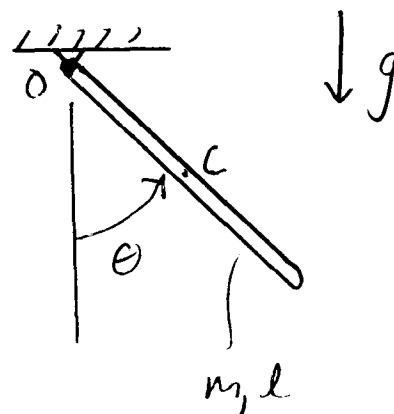
2. Consider the three mass system shown below, consisting of one free mass $m_1 = m$ and two masses, each also of mass m, in a "dumbbell" configuration, that is, connected by a rigid massless bar of length L. The motion described here takes place in a horizontal plane, so that gravity can be ignored. The dumbbell is aligned with the x axis, that is, $\theta = 0$ and its center of mass, C_{db}, is located at (x_{db}, y_{db}). The dumbbell is not rotating and is moving in the $+y$ direction with a constant speed v_o, so that $(\dot{x}_{db}, \dot{y}_{db}) = (0, v_o)$, The isolated mass is sitting at rest at point $(x, y) = (L, 0)$, as shown, so that the system center of mass C is located at $(x_c, y_c) = \frac{2}{3}(L, y_{db})$; note that $y_{db} < 0$ as shown. When the right mass of the dumbbell strikes the isolated mass, an impact occurs with coefficient of restitution e.

(a) Determine the following dynamic quantities at the instant just after the impact:

  (i) The velocity (vector) of the isolated mass, $\vec{v}_1$.

  (ii) The velocities (vectors) of both dumbbell masses, $\vec{v}_2$ and $\vec{v}_3$.

  (iii) The velocity (vector) of the dumbbell center of mass, $\vec{v}_{c,db}$.

  (iv) The angular speed of the dumbbell, taking counterclockwise to be positive, $\dot{\theta}$.

  (v) The velocity of the system center of mass, $\vec{v}_c$.

  (vi) The amount of energy lost during the collision.

(b) Describe in words the resulting motion of the isolated mass and the dumbbell in the limiting cases of perfectly plastic collision, $e = 0$, and perfectly elastic collision, $e = 1$.
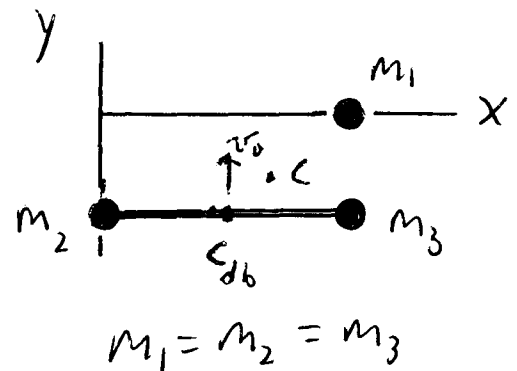
3. A mass-spring system of mass m and total stiffness k is directly forced harmonically at a frequency $\Omega = \sqrt{2k/m} = \sqrt{2}\omega_n$ (note that this is not resonance) in the configuration shown on the left. That is, $F(t) = F\cos\Omega t$. The displacement from equilibrium is x. To reduce the force transmitted to the surroundings through the spring (here the damping element is assumed to be small and can be neglected), an isolation system is to be designed, either by adding mass M as in the middle picture, or by reducing stiffness to the value k_1, as schematically depicted on the right. You can neglect damping by assuming $\zeta = 0$ in your calculations.
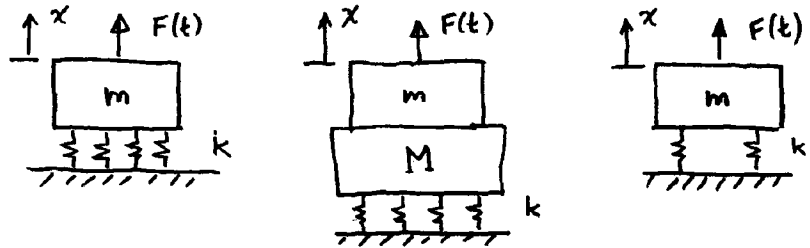
(a) Find the required added mass M such that the dynamic (oscillatory) force transmitted to the surroundings is reduced by a factor of 3.

(b) Find a new value of stiffness, k_1, such that the force transmitted to the surroundings is reduced by a factor of 3.

(c) For both designs (part (a) and part (b)), estimate the amplitude X of the forced vibration response, x. (If you didn't get answers for parts (a) and (b), find the response amplitude of the original system.)

4. A three mass train is schematically depicted below. Damping is not depicted, but is assumed to be light and proportional (or Rayleigh). A force f is applied to the leftmost mass, and f_1 is applied to the rightmost mass. The mass and stiffness matrices are given, as are the forcing vector $\underline{F}$, and two of the modal frequencies ω_i and two of the mode shapes $\underline{u}_i$ (not normalized!).
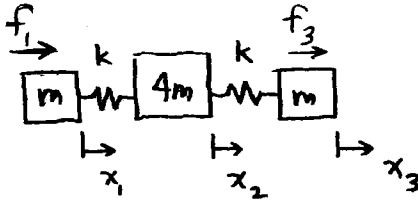
$$\underline{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & 4m & 0 \\ 0 & 0 & m \end{bmatrix}, \quad \underline{K} = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}, \quad \underline{F} = \begin{bmatrix} f_1 \\ 0 \\ f_3 \end{bmatrix}, \quad \underline{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \underline{u}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \omega_1 = 0, \ \omega_2 = \sqrt{k/m}$$

(a) Find the undetermined frequency and mode shape. Show your work.

(b) Is there a rigid body mode? Which mode, and why (mathematically and physically)?

(c) Using the forcing vector $\underline{F}$, determine the value of f_3 in terms of f_1 such that the first mode is not excited.

(d) Write an expression for the forced (steady-state) response of the first mass for the case when $f_1 = -f_3 = F \cos \omega t$. Assume the modal damping of modes 2 and 3 to be ζ_2 and ζ_3, and write your answer in terms of the given parameters.