Student Code Number:_____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.
1. A uniform slender rod of length L is placed at corner B and is released from rest with a vanishingly small initial clockwise rotation. The coefficient of friction at the corner between the surface and the rod is $\mu$.
(A) Find an equation for the angle $\theta$ at which the rod loses contact with the corner.
(B) Use this equation to determine the coefficient of friction that will give $\theta = \text{ArcTan}(4/3)$ for loss of contact.
2. The initially horizontal bar of length $L$ is uniform with mass $M$. It is pinned at $B$ and initially rests also at $A$ with distances as shown. The particle of mass $m$ is moving at speed $v$ with angle $\theta$ with respect to the horizontal. It strikes the bar at end $C$. The impact is frictionless with coefficient of restitution $e$.

(A) Find the angular velocity of the bar immediately after impact.
(B) Find the new angle that the moving particle makes with the respect to the horizontal immediately after impact.

The formulas with your final answers must be concise and as algebraically simple as is possible. Exhibiting formulae in appropriately reduced form is a major aspect of this problem.
3. A motor rests on an elastic platform and is constrained to deflect in the vertical direction. As sketched, it is modeled by a single degree of freedom mass-spring-dashpot system with rotating imbalance. The rotating mass is \( m_r \), and the mass of the block is \( m-m_r \), such that the total mass is \( m \). In this case, \( m = 2m_r \). The undamped natural frequency of the mass/platform assembly is \( \omega_n = 5 \text{ rad/sec} \). When operated (excited) at resonance, the steady-state vibration amplitude is 5 mm. When operated at frequencies over 10 times above resonance, the steady-state vibration amplitude is approximately 1 mm.

(a) Estimate the damping ratio.
(b) Estimate the eccentricity \( e \) of the rotating mass.
(c) Find the mass, \( m_2 \), and stiffness, \( k_2 \), of a vibration absorber tuned at resonance to have a steady-state deflection (of the absorber mass) amplitude of 2 mm, while the motor’s deflection is essentially zero. You can express \( m_2 \) and \( k_2 \) in terms of parameters \( m \) and \( k \).

(Note: in your estimations, you can make small \( \zeta \) (but nonzero) approximations, and for the large operation frequency, you can approximate using the limit of an infinitely large operation frequency.)
4. A mass spring system is depicted below. From left to right, the masses are associated with deflections $x_1, x_2,$ and $x_3$. The equations of motion are, in matrix form,

$$ M\ddot{x} + C\dot{x} + Kx = f, $$

where

$$ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{and} \quad f = \begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{pmatrix} \quad (1) $$

(a) For the case of $m_1 = m_2 = m$ and $m_3 = m$, and $k_1 = 2k$ and $k_2 = k$, the mass, stiffness and damping matrices are given as

$$ M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}, \quad \text{and} \quad K = k \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}. $$

Find the modal frequencies, and the modal vector associated with the largest frequency (no need to normalize for part (a)).

(b) Suppose the system is redesigned slightly, so that $m_1 = m_2 = m$, $m_3 = 2m$, and $k_1 = k$ and $k_2 = 2k$. (i.e. the mass and stiffness matrices are changed.) In this case, the modal frequencies and modal vectors are given as

$$ \omega_1^2 = 0, \quad \omega_2^2 = \frac{k}{m}, \quad \omega_3^2 = 4 \frac{k}{m}, \quad u_1 = \frac{1}{2\sqrt{m}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \quad u_3 = \frac{1}{2\sqrt{12m}} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}. $$

Note that mode 2 is NOT normalized, but modes 1 and 3 are normalized.

Furthermore, the system is proportionally damped such that $C = \alpha M + \beta K$, with $\beta = 0$ (i.e. $C = \alpha M$). Assume that $\alpha$ is small enough (but not negligible) such that all modes are under damped. The system is unforced such that $f = 0$ in equation (1), but has initial conditions such that

$$ x(0) = \begin{pmatrix} 4x_0 \\ 0 \\ -2x_0 \end{pmatrix} \quad \text{and} \quad \dot{x}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. $$

Find the damped free response. Your answer should be in terms of parameters $m, \alpha, k,$ and $x_0$. 

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**Image Diagram Description:**

[Diagram of a mass spring system with three masses labeled $m_1, m_2, m_3$ connected by springs labeled $k_1, k_2$. The masses are associated with deflections $x_1, x_2, x_3$. The diagram includes arrows indicating the direction of displacement from left to right.]