Code Number: ___________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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Directions: Work all four problems.
The problems carry equal weight.
You may use two books for reference.

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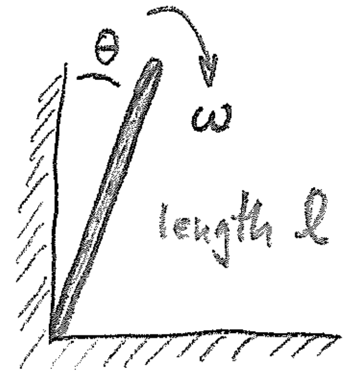
1. The uniform rod is initially straight up against the wall. It is given a small nudge at the top so as to cause it to begin to tip (rotate clockwise) with zero initial velocity. Initially the lower end remains in contact with the corner. There is no friction in this problem. Find:

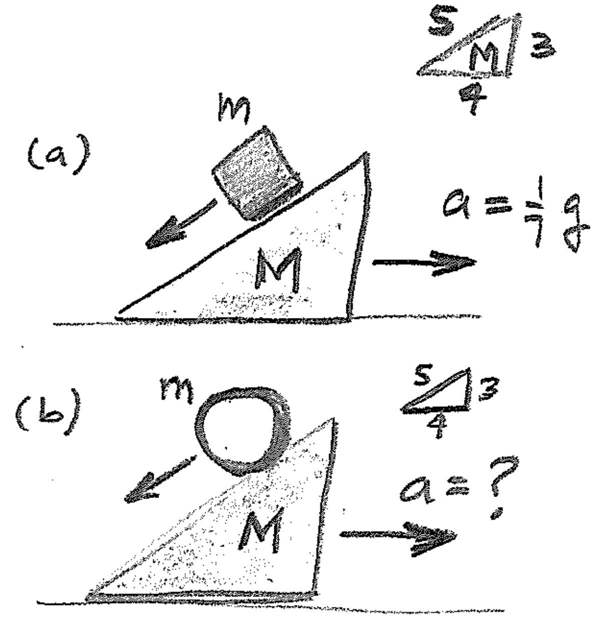
(a) the angle θ at which the lower end begins to move away from the corner;

(b) the rod's angular velocity ω at the instant when the lower end begins to move away from the corner;

(c) the rod's angular velocity ω when it strikes the floor (becomes horizontal).



2. The lower block is of mass M and is free to slide on the floor without friction.
(a) A block of mass m is placed on the inclined surface of the lower block. The bottom surface of this block is very smooth and so it *slides* down the incline without friction. It is observed that the lower block accelerates to the right with acceleration value a = g/7, where g is the acceleration due to gravity. Find the ratio M/m?
(b) The upper block of part (a) is removed and the lower block is restored to rest. A pipe of radius m is placed on the inclined surface of the lower block (the pipe has the same mass as does the sliding mass of (a) but all the mass of the pipe is concentrated at the pipe's radius). The outer surface of the pipe is sufficiently rough that it *rolls without slipping* down the incline. Find the acceleration of the lower block (as a multiple of g)?



3. (six parts) The single degree of freedom system shown in Figure 3.1 below is a model for the vertical motion of trailer supported by leaf springs and shock absorbers. The trailer has mass m_t, carries a load m_ℓ, and its response is measured by the deflection $x(t)$, measured from an inertial frame of reference. The suspension is modeled by the spring k and dashpot c. The trailer has a natural frequency of $\omega_n = \sqrt{k/m}$ where $m = m_t + m_L$ is the total mass, and the trailer system can be assumed to be lightly damped, that is, $\zeta = c/c_{cr} = c/(2\sqrt{km}) << 1$. In the following, subscript u corresponds to the unloaded trailer and subscript ℓ corresponds to the loaded trailer.

For parts (a, b, c) the trailer is unloaded, that is, $m_\ell = 0$, and for parts (c) and (d) you are to consider the same trailer and suspension, but it is now loaded with some $m_\ell \neq 0$.

(a) When the *unloaded* trailer hits a single bump in an otherwise flat road ($y = 0$), it is observed that during five seconds the trailer undergoes four complete cycles and the response amplitude decays by 75% during the time of those four cycles. Estimate values for the natural frequency $\omega_{n,u}$ and ζ_u for the unloaded trailer.

(b) The mass of the unloaded trailer is known to be $m_t = 250$ kg. Determine the spring stiffness k and the damping coefficient c of the suspension.

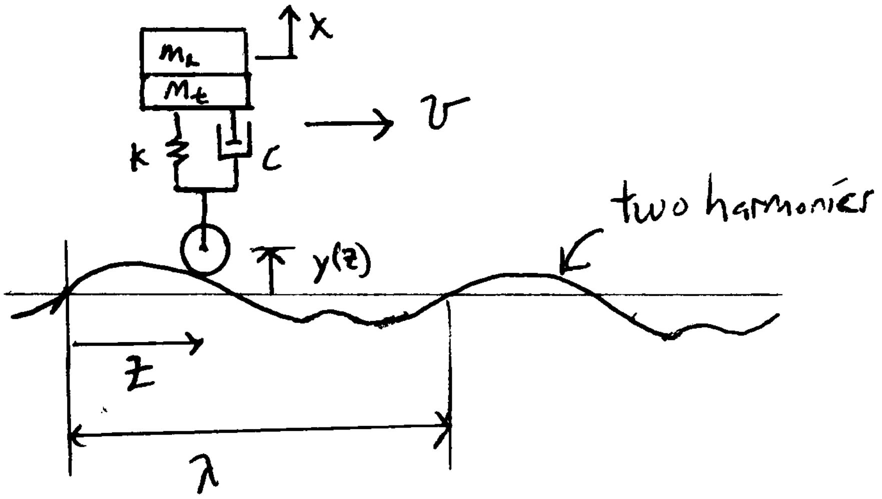
(c) The trailer experiences vibrations when traveling at a constant speed v over a bumpy road, whose profile is idealized by the spatially periodic function $y = Y_1 \sin(2\pi z/\lambda) + Y_2 \sin(4\pi z/\lambda)$, where z ($= vt$) is the horizontal distance along the road, λ is the wavelength of the bumps, and the Y_i's are the amplitudes of the two harmonics of the bumps. For a bumpy road with $\lambda = 10.0m$, determine ALL speeds v for which the unloaded trailer bounce amplitude will be large.

(d) For the trailer loaded with $m_\ell = 400kg$ and the road conditions of part (a), determine the time required for the vibration to decay to 50% of its initial amplitude after hitting the bump.

(e) For the trailer loaded with $m_\ell = 400kg$ and the bumpy road of part (c), some vibration is experienced when traveling at $v = 6.0$m/sec. Should you slow down by 1.0 m/sec or speed up by 1.0 m/sec to reduce the vibration? Give a reason for your answer.

(f) The trailer moving along the bumpy road with a different load from that of parts (c,d) is observed to undergo bad bouncing at speeds of 4.0 m/sec and 2.0 m/sec. Determine the mass of this load, m_ℓ.

Fig 3.1



4. Consider the four degree-of-freedom (DOF) system shown in Figure 4.1 below, which is described by lumped mass and spring elements. The displacements of mass m_i are given by x_i, and f_i represent forces applied to the m_i's, for $i = 1, 2, 3$. The mass and stiffness coefficients are expressed in nondimensional form. The displacement and force column vectors are defined as $\mathbf{x} = (x_1 \ \ x_2 \ \ x_3 \ \ x_4)^{\mathrm{T}}$ and $\mathbf{f} = (f_1 \ \ f_2 \ \ f_3 \ \ f_4)^{\mathrm{T}}$, respectively, where $(\cdot)^{\mathrm{T}}$ denotes the transpose of $(\cdot)$. The inertia and stiffness matrices, in some appropriate units, are given by

$$\mathbf{M} = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{K} = \begin{pmatrix} 7 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix},$$

so that the equations of motion have the form $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}$. The system natural frequencies are given by $\omega_1 = \sqrt{2/5}$, $\omega_2 = \omega_3 = \sqrt{2}$, and $\omega_4 = 2$.

NOTE 1: You can solve these questions without normalizing the modal vectors.

NOTE 2: Some of the following questions can be answered by physical reasoning with very little calculations, and you are welcome, in fact, encouraged, to do the problems this way, provided you give a good explanation for your approach.

(a) The system has a mode shape of $\tilde{\mathbf{u}} = (0 \ \ -1 \ \ -1 \ \ +2)^{\mathrm{T}}$. Determine the natural frequency that corresponds to this mode shape.

(b) The mode shape corresponding to $\omega_1 = \sqrt{2/5}$ has the form $\tilde{\mathbf{u}}_1 = (1 \ \ \gamma \ \ \gamma \ \ \gamma)^{\mathrm{T}}$. Use this fact to construct an equivalent two DOF system for this mode and use it to determine the value of γ.

(c) The natural frequency $\omega_3 = \sqrt{2}$ is repeated, and the mode shapes corresponding to these frequencies have the form $\tilde{\mathbf{u}}_j = (1 \ \ \sigma_1 \ \ \sigma_2 \ \ -1)^{\mathrm{T}}$. Show that $\sigma_1 + \sigma_2 = -2$ must hold for these mode shapes.

(d) Consider the situation in which the system is sitting at rest and the outer two masses $m_1 = 5$ and $m_4 = 1$ are simultaneously struck by identical ideal impulses of magnitude I_o, that is, $\mathbf{f}(t) = I_o\delta(t) \ (1 \ \ 0 \ \ 0 \ \ 1)^{\mathrm{T}}$. Determine the frequency content of the resulting motion, that is, state the frequencies that are present in the response.

(e) Consider the case for which harmonic forcing is applied to $m_1 = 5$ only, that is, the forcing is of the form $\mathbf{f}(t) = (f_o \sin(\omega t) \ \ 0 \ \ 0 \ \ 0)^{\mathrm{T}}$. There will be three values of the excitation frequency ω for which the steady-state amplitude of mass m_1 is zero, that is, $|x_1|_{ss} = 0$. Show that these can be determined from a three DOF system and sketch the appropriate system. One of the modes of this system will have a natural frequency of 2 and a mode shape of $(-1 \ \ -1 \ \ +2)^{\mathrm{T}}$ — describe why this is so.

(f) Suppose that an arrangement of dashpots each with coefficient $c = 1$ are added to the system such that the resulting damping matrix is given by

$$\mathbf{C} = \begin{pmatrix} 19 & -2 & -2 & -2 \\ -2 & 5 & 0 & -2 \\ -2 & 0 & 5 & -2 \\ -2 & -2 & -2 & 8 \end{pmatrix}.$$

Determine if this represents proportional damping and, if so, determine α and β such that $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$.

Fig 4.1