Ph.D. Qualifying Exam:
Dynamics and Vibrations

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1. The uniform bar of length L and mass M is a distance h from the ceiling and originally suspended by means of the two symmetrically disposed inextensible chords. Thus in equilibrium the static tension in each chord is 5/8 M g where g is the acceleration due to gravity. If the right chord is suddenly cut, what is the new tension in the left inextensible chord immediately after the cut?


inextensible chords
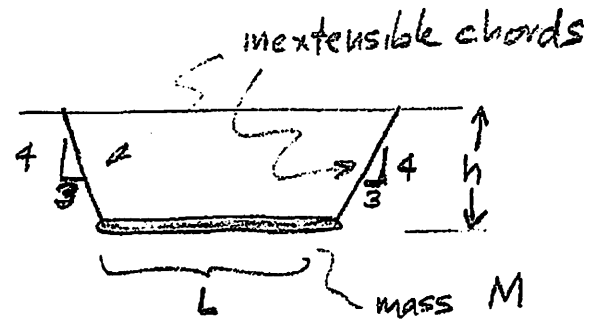
mass M

2. Two barge platforms are floating next to each other and initially in contact in the middle of a lake. Each barge has a mass M when empty. The right barge is carrying a car with mass M/2 (thus the right barge including the car has combined mass M+M/2=3M/2). The left barge is not carrying anything. Initially the whole system is at rest (left barge, right barge and car all have initial velocity zero). Then the car uniformly accelerates to the right for a period of t seconds giving the car a speed of V with respect to the surface of the barge. After this the car brakes are applied for another t seconds bringing the car to rest with respect to the surface of the barge.
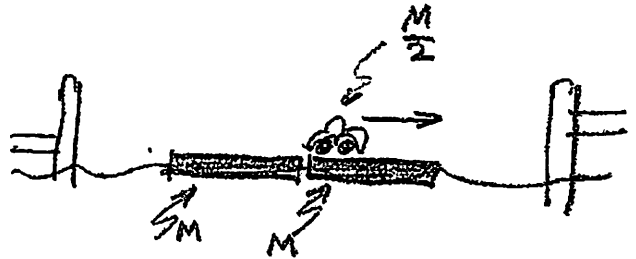
Find the following information after the car is again at rest on the barge surface. Assume that the water only provides a buoyancy force. The barges are initially in contact but are not otherwise linked (they can push against each other, but cannot pull each other).

(A)   the velocity of the left barge (the one without the car).

(B)   The impulse (time integral of the force) that the left shore dock must exert to stop the left barge.

(C)   The velocity of the right barge (the one with the car).

(D)   The impulse that the right shore dock must exert to stop the right barge.

3. (10 pts) To keep the starship from blowing to bits, the Chief Engineer Giordi attaches a 10-kg vibration absorber (m_a = 10 kg) to the engine core. The engine core is externally forced with magnitude F and frequency ω = 20 rad/sec. At this frequency, the engine core is stationary (X_1 = 0), and the absorber amplitude is X_2 = 0.008 m. What is the amplitude of excitation F?
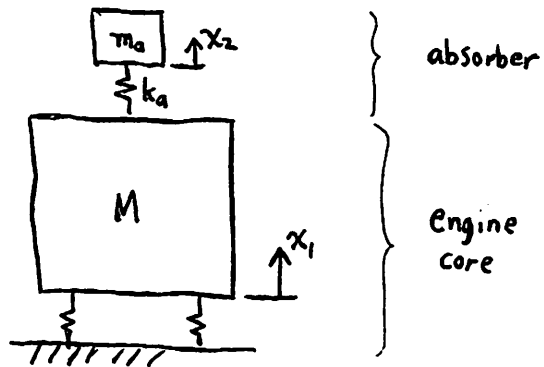
4. (10 pts total) Suppose we have proportional damping in a sinusoidally forced symmetric system. Suppose $M\ddot{\underline{x}} + C\dot{\underline{x}} + K\underline{x} = \underline{F}(t)$, and we have $C = \alpha M + \beta K$, with $\alpha = 0.01$ sec^{-1} and $\beta = 0.02$ sec, and the undamped *normalized modal matrix* P is given below, along with the forcing vector. The undamped natural modal frequencies are $\omega_2 = 2$ rad/sec and $\omega_3 = 3$ rad/sec.

$$P = \frac{1}{\sqrt{m}}\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 2 \end{bmatrix}, \quad F(t) = \begin{pmatrix} 0 \\ 0 \\ f(t) \end{pmatrix}$$

(a) (5 pts) After letting $\underline{x} = P\underline{q}$, write out the differential equation for the third modal coordinate q_3.

(b) (5 pts) Suppose the q_2 equation is worked out and written as

$$\ddot{q}_2 + 2\zeta_2\omega_2\dot{q}_2 + \omega_2^2 q_2 = \frac{3}{\sqrt{m}}\sin\omega t.$$

If the excitation frequency is at the resonance of the second mode, i.e. $\omega = \omega_2$, *approximate* the amplitude X_1 of the coordinate x_1.