Ph.D. Qualifying Exam:
Dynamics and Vibrations

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Directions:
Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two sheets of notes for reference.
No cell phones.

1. A point mass m moves along a horizontal planar surface. A massless inextensible string of total length ℓ passes through a hole in the plane at point O. One end of the string is attached to m and the other to an ideal spring of stiffness k. You are to assume that the string remains in tension at all times, and that the surface and hole are sufficiently smooth so that friction can be ignored in the entire system. To describe the position of m, you are to use coordinates $r(t)$, the length of the string from the hole to m, and the angle $\theta(t)$, as indicated in Figure 1 below. The spring is unstressed when $r = \ell/2$. Your results are to be expressed in terms of parameters and variables $(m,\ \ell,\ k,\ r,\ \dot{r},\ \ddot{r},\ \theta,\ \dot{\theta},\ \ddot{\theta})$. NOTE: The problem is to be solved as a whole, so parts (a) and (b) are not equally weighted.

(a) Determine H_o, the magnitude of the angular momentum of m about point 0, AND show that it remains constant during the motion. (Clearly, the vector $\vec{H}_o$ is normal to the surface.) (b) Using free body diagrams and Newton-Euler methods, determine the (differential) equations of motion that govern $r(t)$ and $\theta(t)$.

Fig. 1.
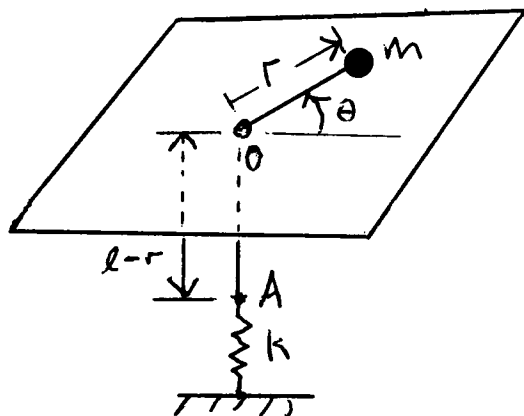
2. A thin uniform bar of mass m and length ℓ is suspended under gravity from a massless slider by a frictionless pin at point O. The bar and slider are initially moving at a constant speed v_o to the right with $\theta = 0$, and $\dot\theta = 0$, that is, the bar is simply hanging straight down, as shown in Figure 2.1 below. At $t = 0$ the slider strikes a stop and instantly stops without any rebound, inducing a swinging motion of the bar described by the angle $\theta(t)$, shown in Figure 2.2 below. Your results are to be expressed in terms of parameters and variables $(m,\ \ell,\ v_o,\ \theta,\ \dot\theta,\ \ddot\theta)$. Note that the bar has center of mass at point C with moment of inertia $I_c = m\ell^2/12$. NOTE: The problem is to be solved as a whole, so parts (a) and (b) are not equally weighted.

(a) Determine the maximum swing angle achieved by the bar, θ_{max}, in the ensuing motion.

(b) Determine the magnitude of the (ideal) impulse, $\mathcal{I}_o$, applied to the slider that causes it to stop at $t = 0$.


Fig 2.1 t < 0

Fig 2.2 t ≥ 0
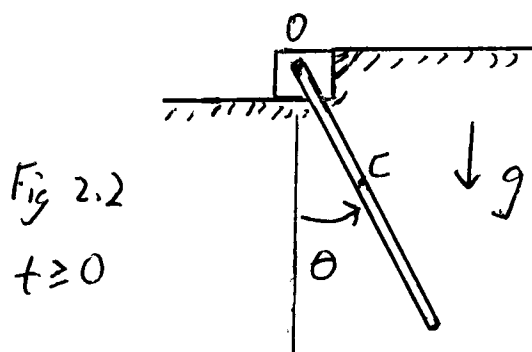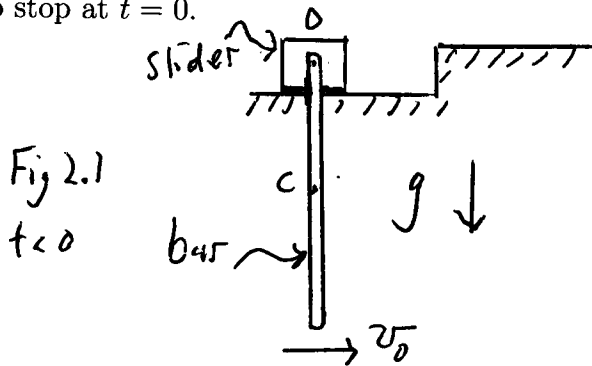
3. A motor assembly has a rotating imbalance quantity $m_r e = 0.001$ kg m (which is the product of the rotating mass m_r and its eccentricity e) and a total mass of $M = 10$ kg. The motor operating speed is $\omega = 30$ rad/sec. A spring support, with negligible damping, is proposed to isolate the vibrations.

(a) For an undamped isolation support, determine the spring constant needed such that the transmitted force is 50% that of the rotating imbalance force.

(b) With this isolation system, the motor has problems on startup. That is, as the motor slowly increases speed, it passes through resonance. It is proposed to add damping. Find the required damping coefficient such that, if held at resonance, the vibration amplitude is held to 0.4 mm.

(c) If the damper is added, will the force transmitted be larger, smaller, or the same, as that transmitted after the undamped design in part (a), when the motor is run at its operating speed? Support your answer.
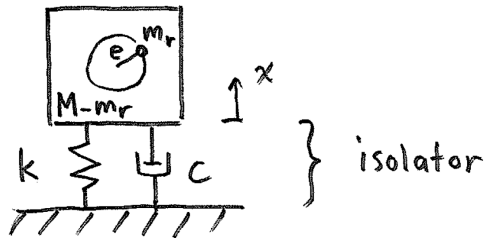
4. The mass and stiffness matrices and the associated mass-spring system are shown. Damping is proportional, such that $C = \alpha M + \beta K$, where $\alpha = 0.2$ s^{-1} and $\beta = 0$. The mode shapes are given (but not normalized), and one of the modal frequencies is $\omega_1 = 0$ rad/sec. Parameter values are $k = 100$ N/m, and $m = 1$ kg. With forcing applied as shown (equal and opposite on the first and third masses), with $F(t) = F_0 \sin(\omega t)$, with $\omega = 10$ rad/sec and $F_0 = 2$ N, find the steady-state response of the third mass (i.e. find the steady-state response of x_3).

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix}, \quad K = \begin{bmatrix} k & -k & 0 \\ -k & 3k & -2k \\ 0 & -2k & 2k \end{bmatrix}, \quad \underline{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \underline{u}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \underline{u}_3 = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$