Department of Mechanical Engineering
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Ph.D. Qualifying Examination
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Dynamic Systems and Control

Open Book
Answer All Questions
All Questions Weighted Equally

Exam Prepared by

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1) Modeling Problem
2) The nonlinear dynamics of the 2011 eco-car (above) are governed by both non-linear Drag $D(t)$ and non-linear tractive $T(t)$ forces.

$$T(t) = T = e[100 - 0.5v] \quad \text{and} \quad D(t) = D = 50 - 0.5v + 0.25v^2$$

For a vehicle mass of 1000kg, the vehicle model is the differential equation

$$m \dot{v} = m \frac{dv}{dt} = T(t) - D(t)$$

yielding the non-linear differential equation model

$$1000 \frac{dv(t)}{dt} + 50 - 0.5v + 0.25v^2 = e[100 - 0.5v]$$

where vehicle model variables are velocity, $v = v(t)$ in kph and throttle, $e = e(t)$ in percent.

a) For the equilibrium operating point at $v_o = 100$ kph, what is the operating point throttle $e_o$ required on percent. (Attach calculations)

$$e_o = \underline{\hspace{2cm}} \%$$

b) Determine the linearized differential equation relating velocity $v(t)$ to throttle $e(t)$ Hint: the linearized equations are written using deviation variables such as $\bar{v} = (v - v_o)$ and $\bar{e} = (e - e_o)$.

(Attach calculations)


c) Determine the steady-state response $\bar{v}_{ss}$ (ft/sec) of the linearized system found above to the constant throttle deviation $\bar{e}_{ss} = 5\%$ from the operating condition found above (Attach calculations).

$$\bar{v}_{ss} = \underline{\hspace{2cm}} \text{kph}$$
3) You are to design a Proportional + Derivative (PD) controller for the system shown below.

\[
\begin{align*}
U(s) & \rightarrow \sum \rightarrow \frac{\text{E}(s)}{K_p + K_D s} \rightarrow \frac{4}{s^2 + 2s} \rightarrow Y(s)
\end{align*}
\]

a) Determine the range of gains \(K_p\) and \(K_D\) that will lead to a stable closed-loop system. (Attach calculations)

b) Find gains \(K_p\) and \(K_D\) that are stable and have a maximum system time constants less than or equal to \(\tau = 0.5\) sec (All others are smaller) and has a closed loop error less than 1%. (Attach calculations)

\[
\begin{align*}
K_p &= \\
K_D &= \\
\end{align*}
\]

b) At the gains you supplied in part b), what is the steady state error for a unit step input to this system? (Attach calculations)
4) Bode Diagram Problem