

Exam Number: _____

Department of Mechanical Engineering
Michigan State University

Ph.D. Qualifying Examination
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Dynamic Systems and Control

Open Book
Answer All Questions
All Questions Weighted Equally

Exam Prepared by

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1) Modeling Problem

Vehicle mass, $m = 1000 \text{ kg}$



2) The nonlinear dynamics of the 2011 eco-car (above) are governed by both non-linear Drag $D(t)$ and non-linear tractive $T(t)$ forces.

$$T(t) = T = e[100 - 0.5v] \quad \text{and} \quad D(t) = D = 50 - 0.5v + 0.25v^2$$

For a vehicle mass of 1000kg, the vehicle model is the differential equation

$$m\dot{v} = m \frac{dv}{dt} = T(t) - D(t)$$

yielding the non-linear differential equation model

$$1000 \frac{dv(t)}{dt} + 50 - 0.5v + 0.25v^2 = e[100 - 0.5v]$$

where vehicle model variables are velocity, $v = v(t)$ in kph and throttle, $e = e(t)$ in percent.

a) For the equilibrium operating point at $v_o = 100 \text{ kph}$, what is the operating point throttle e_o required on percent. (Attach calculations)

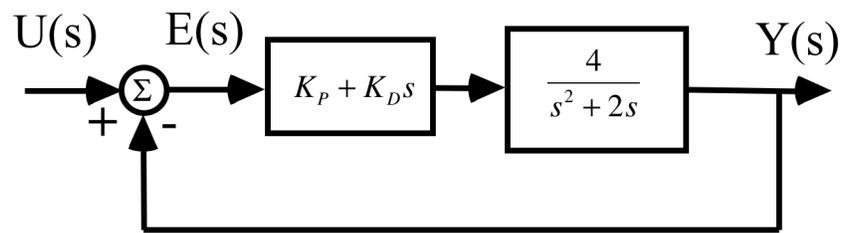
$e_o = \text{_____} (\%)$

b) Determine the linearized differential equation relating velocity $v(t)$ to throttle $e(t)$ Hint: the linearized equations are written using deviation variables such as $\bar{v} = (v - v_o)$ and $\bar{e} = (e - e_o)$. (Attach calculations)

c) Determine the steady-state response \bar{v}_{ss} (ft/sec) of the linearized system found above to the constant throttle deviation $\bar{e}_{ss} = 5\%$ from the operating condition found above (Attach calculations).

$\bar{v}_{ss} = \text{_____} \text{ kph}$

3) You are to design a Proportional + Derivative (PD) controller for the system shown below.



a) Determine the range of gains K_p and K_D that will lead to a stable closed-loop system. (Attach calculations)

b) Find gains K_p and K_D that are stable and have a maximum system time constants less than or equal to $\tau = 0.5$ sec (All others are smaller) and has a closed loop error less than 1%. (Attach calculations)

$K_p =$
$K_D =$

b) At the gains you supplied in part b), what is the steady state error for a unit step input to this system? (Attach calculations)

4) Bode Diagram Problem