1. The input step response of the second-order system

\[ G(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} \]

is shown in the figure below. Determine the values of parameters \( a_0, a_1, \) and \( a_2 \).
Sol:
2. A unit feedback open-loop transfer function is given by

\[ KG(s)H(s) = \frac{K(s + 3)}{s(s - 2)} \]

a) Sketch the root locus of the system as a function of \( K \).
b) Find the value of \( K \) for which the closed loop system is critically damped, and
c) Find the range of \( K \) for which the system is stable without using Routh-Hurwitz criterion.

**Sol:**
3. Consider the following unit feedback system with plant transfer function $G(s)$ and controller transfer function $K(s)$.

$$
\begin{align*}
U(s) & \rightarrow s + 5 \quad \frac{s + 5}{s + 50} \rightarrow 50000 \frac{1}{s(s + 10)(s + 50)} \rightarrow Y(s)
\end{align*}
$$

a) Plot the Body diagram for $G(s)$ (with straight line approximation) in the next page
b) Plot the Body diagram for $K(s)G(s)$ in the next page
c) Calculate the closed loop system phase margin in degree and gain margin in dB

Sol:
4. Consider the following vehicle on a surface with a grade angle $\theta$. Let $v = \dot{x}$ be the vehicle speed, $f_w = c_1 v + c_2 v^2$ be vehicle wind and aerodynamic resistance force, $f_r = c_3 v^2$ be the rolling resistance force, $f_d$ be the vehicle driving force, and $mg$ be the gravity force applied to the vehicle.

![Free Body Diagram of a Vehicle](image)

**a)** Draw free body diagram
**b)** Show that the mechanical system satisfies the following differential equation

$$m \ddot{v}(t) + c_1 v(t) + (c_2 + c_3)v^2 = f_d(t) - mg \sin \theta$$

**c)** Linearize the above nonlinear system at $v_0 = 40$ and $\dot{v}_0 = 0$

**d)** Find linearized transfer function $\frac{\tilde{V}(s)}{\tilde{F}(s)}$, where $\tilde{V}(s) = L[\tilde{v}(t)] = L[v(t) - v_0]$ and

$$\tilde{F}(s) = L\left[\tilde{f}(t)\right] = L[f_d(t) - mg \sin \theta - c_1 v_0 - (c_2 + c_3)v_0^2]$$

**Sol:**