Ph.D. Qualifying Exam

Dynamic Systems and Control

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Open Book
Answer All Questions
All Questions Weight Equally

Time: 3.0 hours
1. Consider the following unit feedback system

$$G(s) = \frac{10k}{s(s+2)}$$

a) Find the closed loop transfer function \( \frac{Y(s)}{U(s)} \).

b) Calculate ramp-error constant \( K_r \) and find \( k \) such that the steady-state error with respect to unit ramp input is 0.5.

c) Find the closed loop system 5\% settling time \( t_s(5\%) \) for \( k = 5 \).
2. Consider the following mechanical system, where $\tau(t)$ is the applied torque; $\tau_d = 10\dot{\theta} + \dot{\theta}^3$ is damping torque; $\tau_{k_1} = 5\theta^2$ and $\tau_{k_2} = 10\theta$ are spring torques; and $J=2$ is the inertia.

![Mechanical System Diagram]

- a) Draw free body diagram
- b) Show that the mechanical system satisfies the following differential equation
  
  \[ 2\ddot{\theta}(t) + 10\dot{\theta}(t) + \dot{\theta}^3(t) + 10\theta(t) + 5\theta^2 = \tau(t) \]

- c) Linearize the above nonlinear system at $\theta_0 = 1$, $\dot{\theta}_0 = 0$, and $\ddot{\theta}_0 = 0$

- d) Find linearized transfer function \( \frac{\hat{\Theta}(s)}{\hat{T}(s)} \)
3. For the closed loop system shown below with the corresponding Root Locus, find the transfer function $G(s)$
4. Draw the body diagram for the given system below with \( k \) equal to 5 and 50 and find the phase and gain margins for both cases

\[
U(s) \quad \begin{array}{c}
\downarrow \\
- \\
\frac{k}{s(s+1)(s+5)} \\
\uparrow \\
Y(s)
\end{array}
\]