Student ID ______________________

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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book and Notes, Formulas are provided on the attached sheet.
- Answer all questions.
- If you think any information is missing, make your assumption then solve. Write down your assumptions.
- All questions have the same weighting.

Exam prepared by

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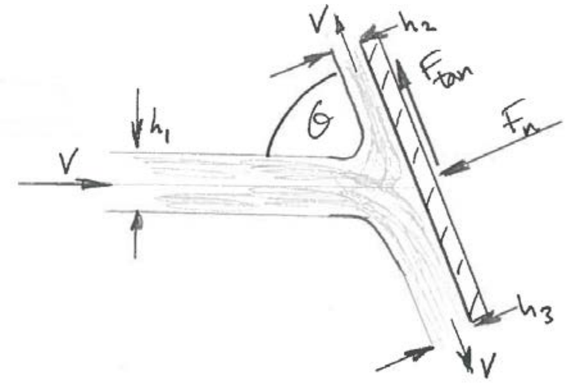
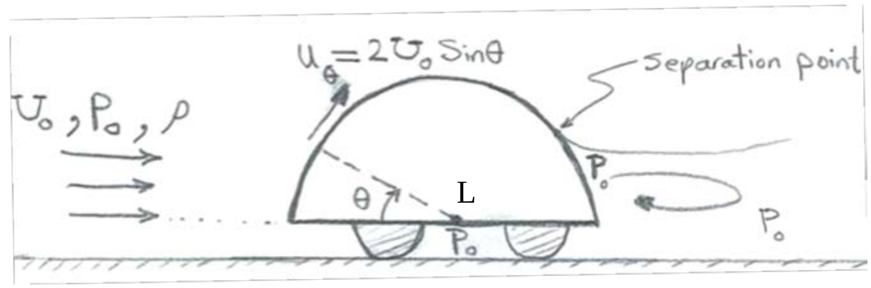
Problem 1: A plane jet of fluid of width $h_1 = 10\,\text{cm}$ and uniform velocity V impinges against the flat plate. It separates into two jets, both of velocity V, and width h_2 and h_3. The tangential force $F_{\tan}$ along the plate is zero and $\theta = 60^0$. Assuming the depth (into the paper) of each jet is 10cm and that gravity acts into the paper, find the values of h_2 and h_3.



Problem 2: The speed of propagation ***C*** of a capillary (very small) wave in deep water is known to be a function only of density ρ, wavelength λ, and surface tension **Y**. Find the proper non-dimensional functional relationship, completing it with a dimensionless constant. For a given density and wavelength, how does the propagation speed change if the surface tension is doubled?

Problem 3: Consider the steady, incompressible, inviscid, and two-dimensional air flow over the following semi-cylinder model for a car. The pressure underneath the car is assumed to be the static freestream pressure $\mathbf{P_o}$. Furthermore, the flow is assumed to be separated at an angle of $\boldsymbol{\theta}$**=150º**, where the pressure is $\mathbf{P_o}$ and remains so in the entire separated region. Find expressions for the lift and drag forces on the car based on the fluid density ρ, freestream velocity $\mathbf{U_o}$, pressure $\mathbf{P_o}$ and car length **L** and spanwise width **W**. Assume the fluid velocity on the upper car surface before the separation point ($0^o < \theta < 150^o$) to be $\mathbf{u_\theta = 2U_o \sin(\theta)}$. Ignore gravity.



Problem 4. As shown in the following figure, two separate films of oil and water are flowing down an infinitely long and infinitely wide thin fixed plate due to gravity that is in opposite **z** direction. The oil and water film thicknesses are of δ_o and δ_w, respectively and the flows in oil and water are fully developed, steady state, incompressible and laminar.

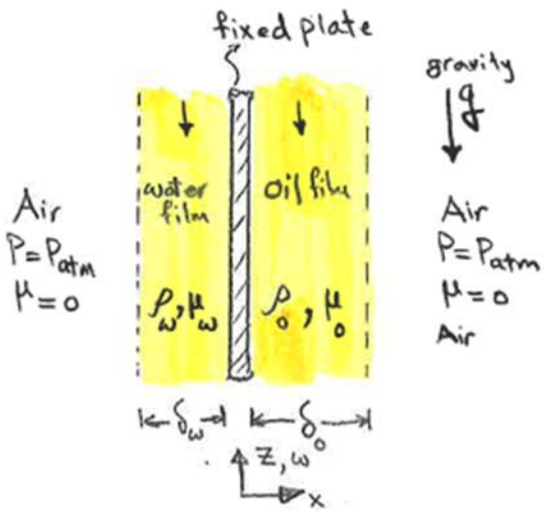
(a) Write the governing equations which describe the velocity field in the water and oil layers attached to the plate.

(b) Write the velocity boundary conditions on water and oil sides.

(c) By assuming that the air pressure is constant and atmospheric everywhere, find the pressure fields in the oil and water layers.

(d) Determine the net force applied by water and oil on the plate as a function of known variables shown on the figure.

Hint: Solve the problem on oil side only. The solution on water side is similar. Just replace oil properties with water properties.



Possibly useful information and formulas:

Bernoulli's Equation: $\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$

Integral equations:

Mass : $0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$

Momentum: $\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho \vec{V}.d\vec{A}$

Differential Equations - Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Differential Equations – Momentums for Incompressible, Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$