Student ID _____

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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book and Notes, Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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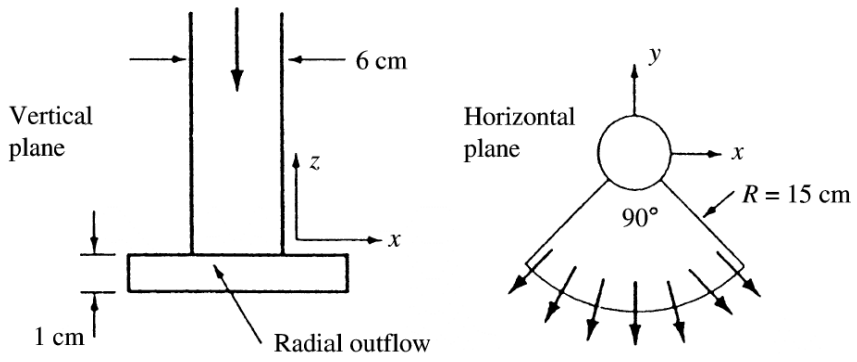
Problem 1: Referring to the figure below, water flows down a vertical 6-cm diameter tube at the rate of 300 gal/min (1 gallon = 0.00378 m^3). The flow then turns horizontally and exits through a 90° radial duct segment 1 cm thick, as shown. If the radial outflow is uniform and steady, estimate the forces (F_x, F_y, F_z) required to support the system against fluid momentum changes. Assume density of water to be 998 kg/m^3.



<u>Problem 2:</u> The lift force F on a rocket is a function of its length L, velocity V, diameter D, angle of attack α, density ρ, viscosity μ, and speed of sound a of the air. Do the following.

(a) Find the relevant dimensionless groups for this problem. Express the interdependence of the dimensionless groups in a function form that allows computation of the non-dimensional lift from other non-dimensional parameters **(form non-dimensional groups that correspond to well-known dimensionless numbers in fluid mechanics)**;

(b) A Saturn V rocket reportedly reaches a speed of 2756 m/s at an altitude of 68 km; where the air properties are as follows:

- Temperature $T = -45°C$;
- Density $\rho = 0.00014$ kg/m^3;
- Dynamic viscosity $\mu = 1.7\times10^{-5}$ Pa.s;
- Speed of sound $a = 300$ m/s.

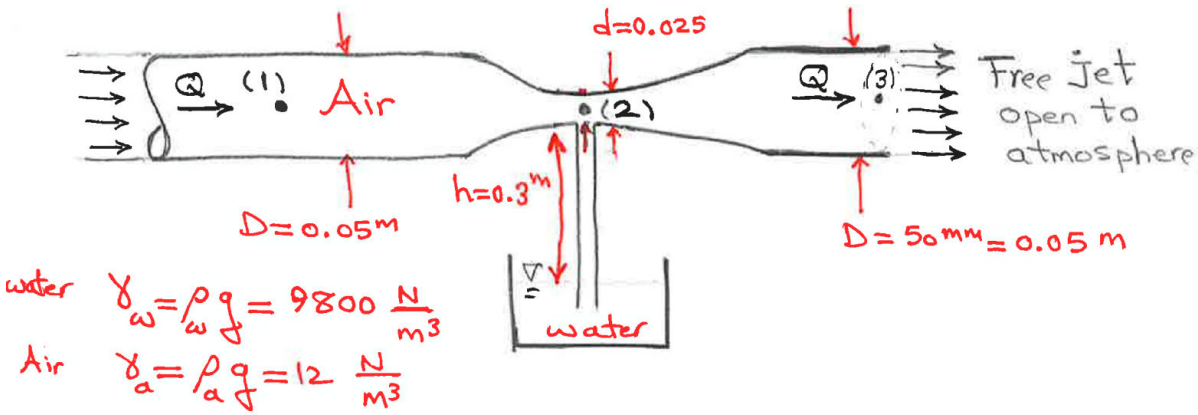
The rocket length is 110.6 m and its diameter is 10.1 m. It is desired to test a 1:100 scaled down model of the rocket in a hypersonic wind tunnel. Assuming the air conditions in the tunnel to be:

- $\rho = 0.04$ kg/m^3;
- $\mu = 1.7\times10^{-5}$ Pa.s;
- $a = 300$ m/s;

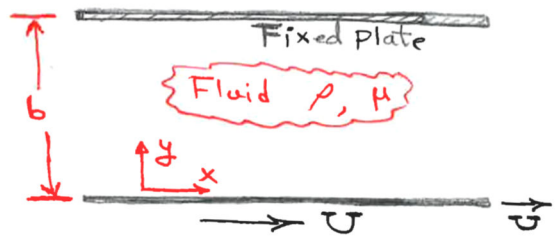
determine:

1. The length and diameter of the model;
2. The test section air velocity for the test. Is complete similarity possible between the model and full-scale rocket? If no, what are the limitations and how would you select the test velocity notwithstanding this limitation?
3. What is the ratio of the lift force measured in the wind tunnel to that encountered by the full-scale rocket (assuming complete similarity)?

Problem 3: Air flows through the device shown in the following figure. If the flow rate in the device is large enough, the pressure within the constriction will be low enough to draw the water up in the tube. Determine the flow rate, Q_1 and pressure, p_1 needed at section 1 to draw water into section 2. Assume steady state flow and neglect compressibility and viscous effects.



<u>Problem 4.</u> The viscous, incompressible flow between the parallel two-dimensional plates shown below is caused by both the motion of the bottom plate with velocity U and the pressure gradient in flow (x) direction, $\Gamma = dp/dx$. Determine the relationship between U and Γ so that the shearing stress acting on the fixed upper plate is zero.



Possibly useful information and formulas:

- Bernoulli's Equation: $\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$

- Pressure: p has units N/m^2 = Pascals. One atmosphere = *1.01325 E+5 Pa*. Force: *1 N = 1 kg-m/s*^{2}. Newton: *1 N = 1 kg-m/s*^{2}*;* Joule (Work) = *N-m*; Watt (Power) = N-m/s. *1 ft = 0.3048 m; 1 in = 2.54 cm; 1 mile = 5280 ft. 1 m*^{3}*= 10*^{3}*L.*

Integral equations:

Mass : $0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$

Momentum: $\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}.d\vec{A}$

Angular momentum: $\sum(\vec{r}\times\vec{F}) = \frac{\partial}{\partial t}\int_{CV} (\vec{r}\times\vec{V})\rho d\forall + \int_{CS} (\vec{r}\times\vec{V})\rho\vec{V}\cdot d\vec{A}$

Differential Equations - Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Differential Equations – Momentums for Incompressible, Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$