Student ID _____

Department of Mechanical Engineering
Michigan State University
East Lansing, Michigan

Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book and Notes, Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

Professor I. S. Wichman
Professor A. Naguib

August 2016

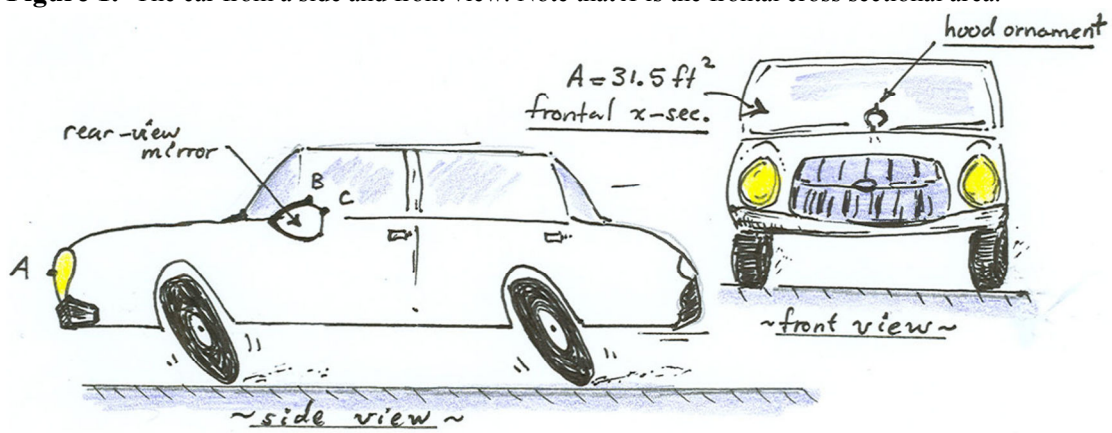
<u>Problem 1:</u> Flow over a car: Consider the automobile shown below in **Figure 1** below. The coordinate system is fixed to the car, which travels with velocity $0 \le U_\infty \le 250\ mph$.

Conversions: *5,280 ft/mile*; *0.3048 m/ft*; *3,600 sec/h*.
Ambient pressure = $1\ atm = 101.355\ kPa$; $\nu_\infty = 1.51 \times 10^{-5}\ m^2/s = 16.25 \times 10^{-5} ft^2/s$.

(a) The automobile makers decide it would be worthwhile to examine the aerodynamic characteristics in a water tunnel. They propose to describe the prototype ($U_\infty = 30\ mph$, $L = 16\ ft$, $\nu_{air} = 1.51 \times 10^{-5}\ m^2/s = 16.254 \times 10^{-5} ft^2/s$) in terms of a full-scale model in water ($L = 16\ ft$, $\nu_{water} = 1.06 \times 10^{-5}\ ft^2/s$). Based on Re matching, what is the required flow velocity? What flow visualization method(s) are typically used in water and provide certain advantages over visualization methods in air? What are these advantages?

(b) The car maximum power is $196.5\ HP = 1.4653 \times 10^5\ W$. The car cross-sectional area is $A = 31.5\ ft^2 = 2.93\ m^2$, the air density is $1.2\ kg/m^3$ and the drag coefficient is $C_D = 0.30$. If the car operates at *80%* maximum efficiency (i.e., only *80%* of the engine horsepower directly produces motion while the rest is lost) what is the maximum speed in *mph*? Use $C_D = F_D/[(1/2)\rho U_\infty^2 A]$.

Figure 1. The car from a side and front view. Note that A is the frontal cross sectional area.



<u>Problem 2:</u> **Shop-Vac:** Under normal operation the 6 HP Shop-Vac motor produces a pressure drop between the atmosphere ($p_{atm} = 1\ atm$) and the inside ducting ($p_{inside} < 1\ atm$), which generates the flow rate $Q = 1\ ft^3/s = 0.0283\ m^3/s$. We take the pressure gradient $(p_{atm} - p_{inside})/L = \Delta p_l = constant$ for this 6 HP motor. The formula for the flow rate is $Q = constants \times \Delta p_l \times R^4$, where R is the hose radius (see **Figure 2**).

(a) How much must the hose radius R be reduced via the accumulation of particles, dust and grime/dirt in order to reduce the Shop-Vac flow rate or "draw" to 50%? The original diameter (see **Figure 2**) is *7.5 cm*.

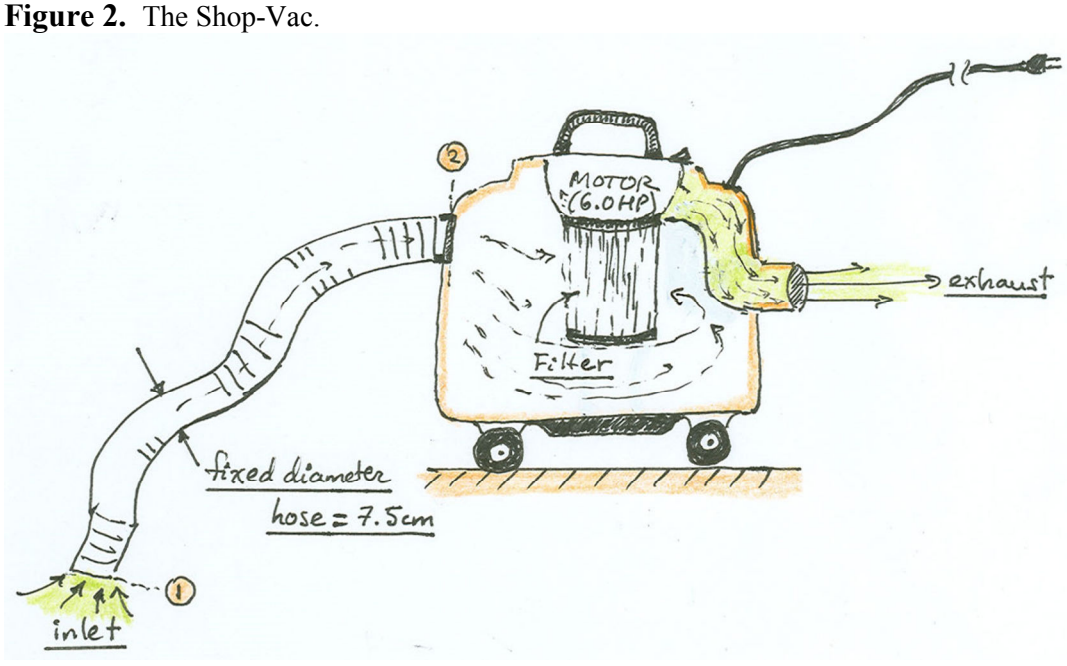
(b) The streamwise (z-direction) steady-flow Navier-Stokes equation is:

$$\rho\left(v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right].$$

Assuming the radial and angular velocities are both zero and that the streamwise velocity varies only in the radial direction (i.e., there is no tumble, no swirl, and the flow is fully developed) and that the streamwise pressure gradient is *constant*, solve the second-order ordinary differential equation for the velocity v_z. Use the boundary condition $v_z = 0$ at $r = R$. Assume μ is constant.

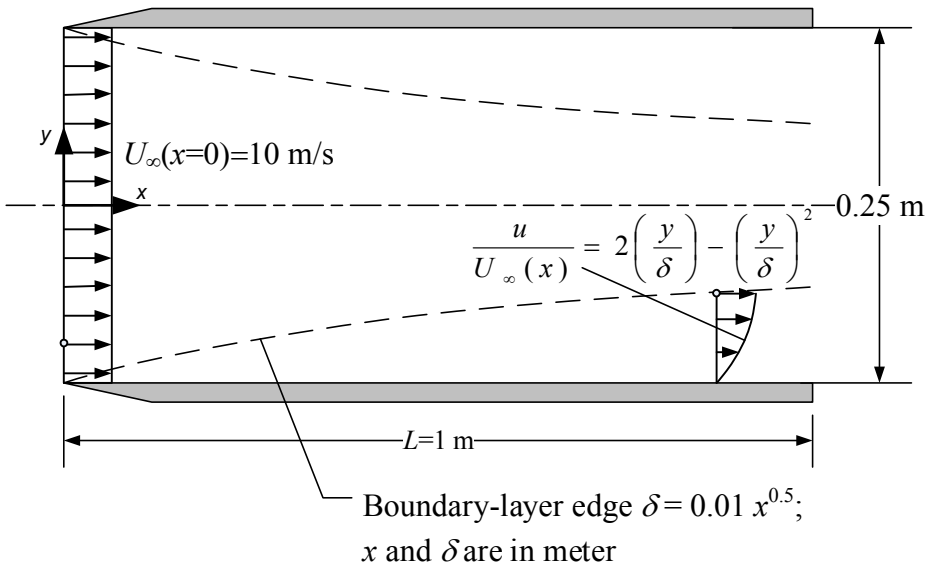
(c) For steady flow draw the streamlines and potential lines for the flow as it enters the hose and as it leaves at the exhaust. Discuss the differences between the two sets of streamlines and potential lines. Identify locations for each case where the flow becomes complicated.

Figure 2. The Shop-Vac.



Problem 3: Uniform, steady, incompressible, inviscid air flow enters the space between two infinitely wide flat plates, as depicted in the figure below. Given the information shown in the figure, assuming the air density to be 1.2 kg/m^3, and making any necessary reasonable assumptions, answer the following:

1. Find the functional variation of the freestream (inviscid) flow velocity U_∞ with x.
2. Find the viscous drag force per unit plate width acting on *each* of the flat walls.



<u>Problem 4:</u> Consider the steady, incompressible, pressure-driven viscous flow between two infinitely wide and long flat plates; as shown in the figure below. The two plates are spaced a distance $2h$ apart. In addition to the main streamwise flow driven by a **<u>constant</u>** pressure gradient dp/dx, flow is injected from the bottom plate and removed from the top plate. The injection and suction velocity at the walls, v_w, is the same and is uniform. **<u>Assuming the flow to be fully-developed, i.e. it does not vary with *x*</u>, show that the streamwise velocity distribution between the plates is given by**:

$$\frac{u}{u_{\max}} = \frac{2}{Re}\left(\frac{y}{h} - 1 + \frac{e^{Re} - e^{Re\frac{y}{h}}}{\sinh(Re)}\right),$$

where

$$Re = \frac{v_w h}{\nu}, \text{ and } u_{max} = \frac{h^2\left(-\frac{dp}{dx}\right)}{2\mu},$$

where, ν and μ are the kinematic and dynamic viscosity respectively. **Demonstrate that in the limit $v_w \rightarrow 0$, the above solution reduces to the familiar parabolic profile of fully-developed 2D channel flow:**

$$\frac{u}{u_{max}} = 1 - \left(\frac{y}{h}\right)^2.$$

Math Reminders:

- $\sinh(y) = \frac{e^y - e^{-y}}{2}$
- A 2nd order linear differential equation with constant coefficients in the form:

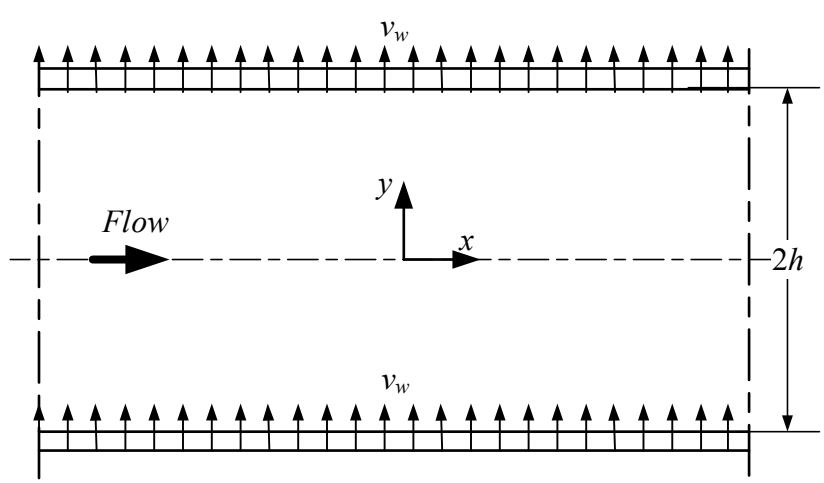
$$\frac{d^2u}{dy^2} + a\frac{du}{dy} + bu = g(y),$$

has a solution of the form $u = u_h + u_p$; where u_h and u_p are the homogenous and particular solutions respectively. u_h has the form $u = e^{my}$, and if $g(y)$ is a polynomial of degree n then u_p is also a polynomial of degree n; specifically:

$$u_p = A_0 + A_1 y + A_2 y^2 + \cdots + A_n y^n,$$

<u>provided</u> that $m \neq 0$ in the homogenous solution. If $m = 0$, then u_p is given by:

$$u_p = y(A_0 + A_1 y + A_2 y^2 + \cdots + A_n y^n).$$



Formula Sheet

Bernoulli Equation:

Bernoulli equation between two points, 1 and 2, on a streamline for steady, inviscid, compressible flow:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Integral equations:

Mass: $0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$

Momentum: $\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}.d\vec{A}$; with $\vec{F}_S$ due to pressure: $\vec{F}_{s,p} = -\int_{CS} p\, d\vec{A}$

Energy: $\dot{Q} - \dot{W}_{CV} = \frac{\partial}{\partial t}\int_{CV}\left(u + \frac{V^2}{2} + gz\right)\rho d\forall + \int_{CS}\left(h + \frac{V^2}{2} + gz\right)\rho\vec{V}.d\vec{A}$

Angular momentum: $\sum(\vec{r}\times\vec{F}) = \frac{\partial}{\partial t}\int_{CV}(\vec{r}\times\vec{V})\rho d\forall + \int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot d\vec{A}$

Differential Equations - Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Differential Equations – Momentums for Incompressible, Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] - \frac{1}{r}\frac{\partial p}{\partial\theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$